*Octonary Numeration, and its Application to a System of Weights and
Measures.   By Alfred B Taylor, A.M., Ph.M.*

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For many years strong and persistent efforts have been made by the
advocates of the French metrical or decimal system, to have its use made
obligatory in the United States, to the exclusion of the heterogeneous
tables of weights and measures now existing. Its use has been legalized
in Great Britain since 1864, and in the United States since 1866.

" On the first of January, 1879, a new Act went into force," (in Eng-
land) " by which it is made unlawful to buy or sell by other than impe-
rial measures, and no provision is made for the adoption of the metric
system."*

Its progress in either country has been very slow.

At the meeting of the British Association for the Advancement of Sci-
ence in 1887, Mr. Ravenstein, of the Geographical section, while strongly
advocating the metric system, stated that " while the English foot is used
by 471 millions of people, the metre is used by only 347 millions of peo-
ple." But the selection of a system evidently should not be made be-
cause a greater number of people use the one or the other, nor on account
of the cost of the change in money or in temporary inconvenience, but it
should be made on the intrinsic merits of the system.

The zealous votary of the metric system can acknowledge no defects ;
the offspring of the world's best science, it must be as perfect as it is beau-
tiful, and only prejudice, ignorance and stolidity can stumble on obstruc-
tions, or refuse entire allegiance to its beneficent sway. The real difficul-
ties in the way of its success are fully realized alone by those who have
given a careful and unbiassed attention, not merely to the various schemes
proposed for simplifying or harmonizing national weights and measures,
but to the practical operation of such reforms when actually applied to the
daily life of human masses. And thus it occurs that what to the enthu-
siast is the foremost virtue of the French system, is, in the view of the
thoughtful student of facts, its most insuperable disadvantage.

The objections to it have been sufficient up to the present time to pre-
vent its adoption, and it is the opinion of very many persons that it can
never be satisfactorily adopted.

Many different projects in remedy of the existing and acknowledged
evils have been suggested ; some more practicable, others more systematic ;
and unfortunately these two classes appear to bear an inverse ratio to each
other.

The substitution of decimal multiples and divisions, conformably to our
established arithmetical notation, has been advocated ; and various stand-
ards or units have been proposed, such as the inch, the foot, the grain, the

* "*New Remedies,*" Vol. viii, p. 192. New York, 1879.

pound, the pint, the gallon, the cubic inch, the cubic foot, etc., but none of these projects has met with much favor.

The most feasible plan for arriving at a satisfactory and authoritative determination of so vital an issue would appear to be the appointment of an international commission, with England, Russia and the Germanic States (with France as well, if practicable), comprising the highest representative talent, not alone from the ranks of the physical philosopher and geometrician, but as well from the classes of merchants, machinists and civil engineers ; from those most interested and most skilled in the subject, for the purpose of organizing and developing an acceptable and permanent system of weights and measures.

Among the labors of such a commission, a very needful one would be to institute a careful and impartial investigation into the exact state and working of the metric system among those nations which had tried it. Assuming nothing, rejecting nothing, accepting nothing, as the ground-work of the future, the commission should endeavor, from a comprehensive survey of all the conditions and all the possibilities involved, to elaborate a scheme best suited to the wants of man, and therefore best entitled to the acceptance of the nations.

If the final verdict were in favor of a uniform octonary system, it would not be difficult to establish it. If, on the contrary, such a commission should agree to adopt the present French system, their decision would go far to silence all further discussion ; the result would be well worth the labor and delay it might cost. No people would receive the system with greater alacrity, or master its details with more facility and promptitude than those of the United States ; not merely from their general intelligence and mental versatility, but from their long training in the use of their decimal monetary system.

Such a conference among nations having so many fraternal ties, seems to be eminently proper in every sense, and surely will not be regarded, at this day, as a visionary or illusive expectation.

The origin of weights and measures is not known, and can be only conjectured. Their need was contemporaneous with the infancy of the human race.

Man in a state of nature would, in his strife for existence, seek food, clothing and shelter from the inclemency of the weather. He would kill animals for their flesh, and use their skins for clothing. The adaptation of skins to this purpose would require measures of some kind to be used. Those naturally suggesting themselves would be the finger, the breadth of the hand, the span, the cubit (or extent from the tip of the elbow to the end of the middle finger), the arm, and the fathom (or extent from the extremity of one middle finger to that of the other, with extended arms). So in the construction of a habitation, however rude, whether of logs, or of earth and stones, he would find need for the use of measures, and some of the above would no doubt supply his needs. Distances traversed in his walks about his habitation would naturally suggest to him measures

of length, and none of those mentioned would conveniently supply his want. Here he would probably use the foot or the pace, and it would not naturally occur to him to use the same measure, or the same scale of pro- portions and numbers to clothe his body and to mark the distance of his walks. Here, then, is a source of diversity in the standards of linear measure, flowing from the difference of relations between man and physi- cal nature. It would be as inconvenient and unnatural to measure a bow and arrow, for instance (among the first implements of solitary man), by his foot or pace, as to measure the distance of a day's journey, or a morn- ing's walk to the hunting ground, by his arm or hand. These natural standards are never lost to individual man in any stage of society. There are probably few persons living who do not occasionally use their own arms, hands and fingers to measure objects which they handle, and their own pace to measure a distance upon the ground.

The need of measures of capacity would not be felt at quite so early a period of man's history as measures of length, yet they would be rendered necessary by the nature of liquids, and for the admeasurement of those substances which nature produces in multitudes too great for numeration, and too minute for linear measure ; of this character are all the grains and seeds, which from time to time, when man becomes a tiller of the ground, furnish the principal materials of his subsistence. But nature has not furnished him with the means of supplying this want, in his own person, and as his first measures of capacity he would probably employ the egg of a large bird, the shell of a mollusk, or the horn of a beast. The want of a *common* standard not being yet felt, these measures would be of vari- ous dimensions ; nor is it to be expected that the thought would ever occur to the man of nature, of establishing a proportion between the size of his arm and his cup, of graduating his pitcher by the size of his foot, or equalizing its parts by the number of his fingers. The necessity for the use of weights comes still later. It is not essential to the condition or comforts of domestic society. It presupposes the discovery of the prop- erties of the balance ; and originates in the exchanges of traffic after the institution of civil society. It results from the experience that the com- parison of the articles of exchange, which serve for the subsistence or the enjoyment of life, by their relative extension, is not sufficient as a crite- rion of their value. The first use of the balance and weights implies two substances, each of which is the test and standard of the other. It is nat- ural that these substances should be the articles most essential to subsist- ence. They will be borrowed from the harvest and the vintage ; they will be corn and wine. The discovery of the metals, and their extraction from the bowels of the earth, must, in the annals of human nature, be subsequent, but proximate, to the first use of weights ; and when dis- covered, the only mode of ascertaining their definite quantities will soon be perceived to be their weight. That they should themselves immedi- ately become the common standards of exchanges, or otherwise of value and of weights, is perfectly in the order of nature ; but their proportions to

one another, or to the other objects by which they are to be estimated, will not be the same as standards of weight and standards of value. Gold, silver, copper and iron when balanced each by the other in weight will present masses very different from each other in value. They give rise to another complication, and another diversity of weights and measures. The balance, or scales, in a rude form, are known to have been in use from very early times. The Greeks, as appears from the Parian chronicle, believed weights, measures, and the stamping of gold and silver coins to have been alike the invention of Phidon, ruler of Argos, about the middle of the eighth century B. C.

The weights or counterpoises used in weighing were probably obtained by taking equal bulks, roughly determined, of some material of comparatively uniform density, such as brass or iron ; but to render them more accurate and definite it became necessary to call in the aid of more accurate measures of capacity ; and the weight of a known volume of pure water, at a known density, is now the criterion universally resorted to for determining the standard of weight. This supposes that the volume or cubic contents are correctly known ; and since contents or capacity can be practically expressed only in terms of the cube of a length, and area in terms of the square of a length, it follows that to obtain exact units of measure of all kinds, it is necessary first to fix, and then to be able to reproduce with the greatest possible exactness, the unit of length. Absolutely invariable standards of weight and measure have not been, and in the nature of the materials to be dealt with, cannot be attained ; while to secure and reproduce measures of given sorts, the results of which shall be correct and uniform to within the least practicable degree of variability, is a problem upon which a vast amount of scientific research, ingenuity and labor has been expended.

When the legislator has the subject of weights and measures presented to his contemplation, and the interposition of law is called for, the first and most prominent idea which occurs to him is that of uniformity ; his first object is to embody them into a system, and his first wish to reduce them to one universal common standard.

In England, from the earliest records of parliamentary history, the statute books are filled with ineffectual attempts of the legislature to establish uniformity.

Of the origin of their weights and measures, the historical traces are faint and indistinct ; but they have had from time immemorial, the *pound, ounce, foot, inch* and *mile,* derived from the Romans, and through them from the Greeks, and the *yard,* or *girth,* a measure of Saxon origin, but as a natural standard different from theirs, being taken not from the length of members, but from the circumference of the body, and hence a source of diversity. The yard, however, very soon after the Roman conquest, is said to have lost its original character of girth ; to have been adjusted as a standard by the arm of King Henry the First ; and to have been found

or made a multiple of the foot, thereby adapting it to the remainder of the system.

In 1266, the first positive attempt was made to change the common weight into the troy,* under the name of the weight of assize ; a statute 51. Henry III enacted "that an English penny called a sterling round, and without any clipping, shall weigh 32 grains of wheat, from the middle of the ear, and 20 pence to make an ounce, 12 ounces a pound, 8 pounds a gallon of wine, and 8 gallons of wine a bushel of London, which is the eighth part of a quarter." This penny weight was divided into 24 grains.

But neither the present avoirdupois, nor troy weights, were then the standard weights of England. The foundation of the system of 1266 was the penny sterling, which was the 240th part of the *tower pound ;* the sterling or easterling pound which had been used at the mint for centuries before the conquest, and which continued to be used for the coinage of money until the eighteenth year of Henry the Eighth, 1527, when the troy pound was substituted in its stead.   The tower pound weighed 360 grains (or $\frac{1}{15}$) less than the pound troy, and the penny, therefore, weighed $22\frac{1}{2}$ grains troy.

The philosophers and legislators of Britain have never ceased to be occupied upon weights and measures, nor to be influenced by the strong desire for uniformity. They found a great variety of standards differing from each other, and instead of searching for the causes of these varieties in the errors and mutability of the laws, they ascribed them to the want of an immutable standard from nature. They felt the convenience and the facility of decimal arithmetic for *calculation ;* and they thought it suscep- tible of equal application to the divisions and multiplications of *time*, *space* and *matter*. They despised the primitive standards assumed from the stature and proportions of the human body. They rejected the sec- ondary standards taken from the productions of nature most essential to the subsistence of man ; the articles for ascertaining the quantities of which weights and measures were first found necessary. They tasked their ingenuity and their learning to find, in matter or in motion, some *immutable* standard of linear measure which might be assumed as the sin- gle universal standard, from which all measures and all weights might be derived. In France their results have been embodied into a great and beautiful system.   England and America have been more cautious.

Among the earlier measures of length used by various nations are found such as the "finger's length," the "digit" (second joint of the forefinger), the "finger's breadth," the "palm," the "span," the "cubit" (length of forearm), the "nail," the "orgyia" (stretch of the arms), the "foot," the "pace," etc., and the names of these measures,

* When the troy weight was introduced into England is not known. It was introduced into Europe from Cairo in Egypt about the time of the Crusades, in the 12th century. Some suppose its name was derived from *Troyes,* a city in France, which first adopted it ; others think it was derived from *Troy-novant,* the former name of London.

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their almost constant recurrence among different nations, and the close approximation in length of such as have, like the foot, more nearly acquired the character of arbitrary measures, alike establish the fact that in its origin, measurement of length was by the application of parts of the human body. In some parts of the East the Arabs, it is said, still measure the cubits of their cloth by the forearm, with the addition of the breadth of the other hand, which makes the end of the measure ; and the width of the thumb was in like manner formerly added at the end of the yard by the English clothiers. The advantages of such measures for popular use are that they are known by observation and readily understood, and in an average way always capable of being recovered, when more arbitrary standards might be wholly lost. But their great disadvantage is extreme variableness, especially when directly applied ; and in the gradual progress of men's minds toward exactness of conception and reasoning, three successive plans of insuring greater accuracy have been devised, and two at least have secured permanent adoption.

The first is that of obtaining a uniform standard by exchanging the measures by parts of the body for conventional or arbitrary lengths, which should represent the average, and which were to be established by law.

The second plan is that of making accurate comparisons of the various standards of each given sort in a country. Attempts of this kind appear in England to have been commenced under the auspices of the royal society in 1736 and 1742 ; in the former year by a comparison of the English, French and old Roman standards ; and in the latter by the determination (by George Graham) of the length of a pendulum beating seconds at London, to be equal to 39.1393 inches, and the construction of a standard yard. Of this, under the direction of the House of Commons, Mr. Bird (a celebrated optician) prepared two accurate copies, respectively marked "standard yard 1758" and "1760," and intended for adoption as the legal standards. He determined and prepared also the pound troy, the original of that now in use. Of these two standards, no intentional alteration has since been made ; so that these or their derivatives are now in use in England and the United States.

The third proposed step toward rendering measures exact has reference rather to the means of making the standards recoverable in case they should be lost. In the definite pursuit of this purpose the French philosophers of the time of the Revolution took the lead, and devised the metric system, in which the unit of length is derived from the dimensions of the earth, and the units of capacity and weight are made dependent upon the former, while the whole has decimal multiples and subdivisions. The celebrated commission concentred within itself the physical and mathematical science of France, but there was one science unfortunately not there represented ; the science of human nature. Looked at from a purely arithmetical standpoint, the problem of measures suggested but one solution, that of the decimal digits. Abstract mathematics could furnish no inducements to binary or octonary divisions or progressions.

So early in our national existence as the year 1790, the illustrious Jefferson, then Secretary of State, in obedience to a resolution of Congress calling upon the Secretary to propose a plan or plans for establishing uniformity in the currency, weights, and measures of the United States, presented a report recommending a decimal system of metrology, and its derivation from a natural and permanent standard of length.

Instead of taking the ordinary pendulum of 39 inches, he proposed the second's rod of 5 feet, then generally known as Leslie's pendulum rod. A simple straight rod, without the bob or ball, suspended at one end, has, as is well known, its centre of oscillation at a distance of two-thirds of its length from its point of suspension ; or, in other words, is one-half longer than the common loaded pendulum vibrating in the same time. Such a rod vibrating seconds is 58.72368 inches long ; dividing this into five equal parts, Mr. Jefferson took this fifth part, or 11.744736 inches as the length of the new '' foot,'' and from this by decimal multiples and subdivisions he presented a series of tables of weights and measures.

When we reflect that the system of metrology here displayed was perfected by Mr. Jefferson before any steps had been taken by the French government toward the decimal re-organization of weights and measures in that country, we must regard it as a memorial in the highest degree creditable to the judgment and contriving skill of its author ; and as one of many illustrations of the varied activity of his mind, and of the interest he ever felt in all schemes for human improvement. The great superiority of his proposed scales of measure, to those in common use, cannot be questioned ; and their adoption would have been a signal public benefit. The tables presented by him form a connected and complete system, each depending directly upon the one preceding, and necessarily flowing out of it, and all determined from a single and invariable natural standard by a very simple and beautiful mode of derivation.

In this respect, however, the French system is by far the best of all that have yet been devised. Starting with a carefully measured quadrant of the earth's meridian, and dividing it into ten million parts, this system presents us with a '' metre ''* as a universal standard to which all others may be referred. Indeed, if a decimal system of weights and measures is to be ultimately adopted, there appears to be none that has such just claims to our acceptance as that of the French ; and although it would be much more difficult of popular introduction than a simple decimalization of our own divisions, and therefore less '' practicable,'' there can be no doubt that it would be in every way superior, both in regard to the precision of its measures, and the simple and philosophical character of its divisions ; besides all which it has the immense advantage of being already introduced and in successful practical operation throughout the great Republic of France ; and every extension of its use would be an important step in the progress toward a uniform system among all nations.

* Equal to 39.370788 inches ; very nearly the length of the second's pendulum, and not much longer than our yard.

Beautiful and simple as this system appears, and clear as its nomenclature is to those familiar with the Greek and Latin tongues, it is yet open to animadversion on practical grounds, in that its language is that of the philosopher, and not of the tradesman or the business man. To all but classical scholars—that is, to the large majority of men—the terms used in the French tables are difficult and unmeaning; to be acquired and appreciated only by a laborious effort of abstract memory, and even when thus acquired, constantly liable to be confounded and mistaken. Its metres and litres, its myriametres and myrialitres, its decigrammes and decagrammes, are admirably contrived to bewilder the uninitiated, but of all possible devices are the least adapted to the common uses of daily life. To obtain a ready and direct apprehension of the values of different denominations of measure, it is necessary that each should be recognized as an independent unit, without reference to its fractional or multiple derivation. Thus, "ounces" or "inches" are at once seized upon by the mind as distinctive standards of value; and the fact that these terms both signify "twelfths" (being derived from the Latin "uncia") never enters into our contemplation when using them. The coin a "cent" has come to signify a "one" and not a "hundredth." What is really needed then for the popular service, is a set of names, brief, easy, and distinctive by a wide separation of sound, however arbitrary or unmeaning may be their origin. In this view of the matter, the rude and indefinite vulgarisms of "grains" and "scruples," "feet" and "rods," "gills" and "gallons" are infinitely preferable to the scientific jargon of *centigrammes* and *milligrammes,* and *hectogrammes* and *kilogrammes.* In fact, the French system has totally ignored all units, excepting the single one selected as the standard for each table. Thus in weight, the French cannot be said to have any other measure than the gramme; and instead of resorting to the dead languages for so familiar a thing as a simple numeration table, it would be much better to speak of and write down, the multiples or divisions of this weight as a thousand or a hundred grammes, or as so many hundredths or thousandths of a gramme. This, in plain English (or plain French), would be understood by every one, and would just as conveniently express everything that is contained in the high-sounding terms we have characterized as "scientific jargon."*

An almost unmanageable difficulty in the introduction of the French

* While thus strongly expressing our objection to the *nomenclature* of the French tables (whose very fault is its excess of system), it would be unjust not to acknowledge, and ungenerous not to admire, the catholic sentiment which dictated it. The eminent philosophers to whom belongs the honor of developing a metrology by far the most perfect that has yet been devised, felt as if they were legislating for the civilized world. Desirous that all might have the benefit of their labors, they rejected all the familiar terms employed in France, and naturally resorted to the great storehouse from which the scientific world has ever been accustomed to draw its technical phraseology; exhibiting in this, their anxiety to adopt a language which might be acceptable to all nations. Unfortunately it is suited to none. The language of science cannot be that of the shop and the market-place.

system has been found in the adoption of the nomenclature ; there is a natural aversion in the mass of mankind to the adoption of words, to which their lips and ears are not from their infancy accustomed. Hence it is that the use of all technical language is excluded from social conversation, and from all literary composition suited to general reading ; from poetry, from oratory, from all the regions of imagination and taste in the world of the human mind. The student of science in his cabinet easily familiarizes to his memory and adopts without repugnance words indicative of new discoveries or inventions, analogous to the words in the same science already stored in his memory. The artist, at his work, finds no difficulty to receive or use the words appropriate to his own profession. But the general mass of mankind shrink from the use of unaccustomed sounds, and especially from new words of many syllables.

Should these measures be therefore introduced, we should strongly urge the entire abolition of the French nomenclature, and the complete naturalization of the different scales by the substitution of more familiar terms from our vernacular tongue.

In the advancement of physical science no nation has taken a higher position, or exhibited a more fertile activity, than France. Hence it has become necessary for every English and American physicist to familiarize himself with the French units and standards of scientific research and discovery, if he would avail himself of their benefits or information. This again has induced a considerable employment of the same scales by the English and American *savants*, in repeating or extending the foreign experiments. It is not remarkable, therefore, that the scientific world generally, both in this country and in England, should desire to see this system universally prevail. Very few scientific men have given the subject of popular weights and measures any special attention, and of those who have, it is believed that a very small proportion will be found to advocate the unqualified adoption of the metric system.

A decimal system applied to weights and measures must result in failure as regards the convenience of such a system or its adaptation to popular wants, and this want of adaptation arises, not from any defect in the plan on which it is established, but from inherent defects in the decimal system of numeration.

The introduction of any new system of weights and measures, to take the place of one long established and in general use, will be found a troublesome and difficult exercise of legislative authority. There is indeed no difficulty in enacting and promulgating the law, but the difficulties of carrying it into execution are always great.

Of all the difficulties to be overcome, however, perhaps the greatest is the abandonment of old and familiar units or standards.

"Weights and measures may be ranked among the necessaries of life to every individual of human society. They enter into the economical arrangements and daily concerns of every family. They are necessary to every occupation of human industry ; to the distribution and security of

every species of property ; to every transaction of trade and commerce ; to the labors of the husbandman ; to the ingenuity of the artificer ; to the studies of the philosopher ; to the researches of the antiquarian ; to the navigation of the mariner, and the marches of the soldier ; to all the ex changes of peace, and all the operations of war. The knowledge of them, as in established use, is among the first elements of education, and is often learned by those who learn nothing else, not even to read and write. This knowledge is rivetted in the memory by the habitual application of it to the employments of men throughout life. Every individual, or at least every family, has the weights and measures used in the vicinity and recog nized by the custom of the place. To change all this at once, is to affect the well-being of every man, woman and child in the community. It en ters every house, it cripples every hand.''

The failure that attends the introduction, and the objections that have so far prevented the adoption of the metric system in Great Britain and in the United States, notwithstanding the strenuous and untiring efforts of its advocates, sufficiently attest the need of some other scheme, which, while possessing the advantages claimed by that, may be free from its dis advantages and defects.

Great Britain has shown such a determined opposition to the metric sys tem, that, in the International Monetary Conference held in Paris in 1867, she refused even to negotiate in reference to unity of coinage, and her dele gates stated '' that until it should be incontestably demonstrated that the adoption of a new system offered superior advantages justifying the aban donment of that which was approved by experience and rooted in the hab its of the people, the British government could not take the initiative in assimilating its money with that of the Continent.''

She maintains the most complex system of measures, weights and coin age now in use among civilized nations ; she persistently rejects the deci mal system and adheres to the complex division of pounds, shillings and pence, a system abandoned by the United States in their rejection of col onial dependence.

A very strong objection to accepting the metre, either directly or indi rectly, as our national standard of length is the want of absolute precision in the rule itself. It has been shown by the investigations of able mathe maticians, that the metre is not an exact expression of its theoretical value, and as the result of more extended geodetic measurement up to 1875, that the quarter of the meridian is equal to 10,001850 metres, and that consequently the metre is too short by $\frac{1}{5400}$ part of its length. This unfortunate and vital defect in the French metre nullifies almost en tirely its value as a natural standard, and defeats the principal object of its establishment—the facility of its perfect restoration in all future time should the existing material standards be destroyed. The metre is just as arbitrary a standard as the yard ; the only real thing about it is the plat inum rod in the public archives in Paris, and this has no advantage over the English standard kept in the British exchequer.

The kilogramme has in like manner been found to differ from its assumed value by some small fraction, in consequence of the great difficulty attending exact determinations of this kind.

Our weights, measures and coins at present correspond much more nearly with the English than with the French standard. Our commerce with Great Britain is very much greater than with any other nation, and we should certainly commit a great error in adopting the metric system unless Great Britain should consent to adopt it also.

Our adoption of the metric system, and the consequent change of our linear unit, would sever our uniformity with Great Britain, a country with which perhaps three-fifths of our foreign commerce is transacted, besides which it would entail great inconvenience and much greater expense than is generally imagined. The measurements of every plot of ground in the United States have been made in acres, feet and inches, and are publicly recorded with the titles to the land according to the record system peculiar to this country. What adequate motive is there to change these expressions into terms which are necessarily fractional, and in which those foreign nations, whose convenience it is proposed to meet, have no conceivable interest? What useful purpose is subserved by designating a building lot 20×100 feet in the form $6.095889 \times 30.479448$ metres?

Besides this, the industrial arts during the last fifty years have acquired a far greater extent and precision than were ever known before. Take, for instance, the machine shop, in which costly drawings, patterns, taps, dies, rimers, mandrils, gauges and measuring tools of various descriptions, for producing exact work, and repetitions of the same with interchangeable parts, are in constant use. It has been calculated that in a well-regulated machine shop, thoroughly prepared for doing miscellaneous work, employing two hundred and fifty workmen, the cost of a new outfit adapted to new measures would be not less than one hundred and fifty thousand dollars, or six hundred dollars per man.*

Supposing full consent were obtained for using metric measures in all new machinery, how slow and difficult would it be to make the change. A very large proportion of work consists in renewing worn parts; where, then, are the new measures to come in? The immense plant of railway motive power in the United States is all made to inches and parts. At what time can a railway company afford to change the dimensions of the parts of a locomotive engine? At no time, because the change would require to be simultaneous in the whole stock. It is true that the old dimensions might be adhered to, and called by metric names, putting 0.0254 metres, or 25.4 millimetres for one inch; but this would be only an evasion, not a solution of the problem.

A practical defect in the working of this system, which has been demonstrated by experience, is its incapability of binary divisions; a defect which of course attaches equally to every decimal scale; and one which

* "The Metric System in our Workshops," etc., by Coleman Sellers. Journal of the Franklin Institute, Philadelphia, June, 1874.

with great precision into halves, quarters, and eighths, while we should have to make repeated trials to divide the same into thirds or fifths, and then attain the result only tentatively and approximately. And lastly, it appears to be the most natural of scales, from the very common use of the two hands in separating objects into pairs.*

Such being the claims, then, of the binary scale of geometrical progression, and such its obvious advantages over all others, it is not surprising that this should be found to be practically the prevalent mode of distributing the more common weights and measures throughout the world, whatever may be the multiples or divisions enacted by law.

The Roman weights in general use throughout the empire (that is, throughout the civilized world) for some centuries after the Christian era, were by means of intermediate subdivisions (introduced by the common consent of traders) practically distributed upon a binary scale. So with the divisions in universal use at the present day ; we find that a nest of avoirdupois weights comprises $\frac{1}{4}$ oz., $\frac{1}{2}$ oz., 1 oz., 2 oz., 4 oz., 8 oz. and 16 oz., or 1 pound, and sometimes a 2-pound weight and a 4-pound weight ; and by this scale of binal progression, almost everything is purchased at retail. Our yardsticks are found to be divided not into the legal feet and inches, but into halves, quarters, eighths and sixteenths. Precisely so with the inch, which is never divided into its primitive "three barleycorns," but almost always, like the yard, by the binal scale into eighths and sixteenths, though occasionally divided for particular purposes into twelfths, or into tenths. The operation of this great law is quite as strikingly exhibited in France, where the popular necessities have compelled the introduction of binal divisions, not recognized by the established decimal scales, nor, indeed, strictly compatible therewith.

Mr. Peacock, in his admirable treatise on "Arithmetic," in the *Encyclopedia Metropolitana*, thus sums up his review of the French system : "The decimal subdivision of these measures possessed many advantages on the score of uniformity, and was calculated to simplify, in a very extraordinary degree, the arithmetic of concrete quantities. It was attended, however, by the sacrifice of all the practical advantages which attend subdivisions by a scale admitting of more than one bisection, which was the case with those previously in use ; *and it may well be doubted whether the loss in this respect was not more than a compensation for every other gain.*" This deliberate judgment is from the author of perhaps the

* "The classification by pairs which nature points out would suggest the simplest mode of reckoning. Counting these pairs again by two, and repeating the procedure, we arrive by progressive steps at the radical terms, 4, 8, 16, etc." (*Edinburgh Review* for May, 1811, Vol. xviii, p. 185).

The celebrated Leibnitz, so eminent as a mathematician as well as a philosopher, struck with the simplicity and peculiar capabilities of this scale, proposed and strongly urged the introduction of Binary Arithmetic. He showed that the Binary system, in addition to its extreme facility, possessed peculiar value in discovering the properties of numbers, and in constructing tables, etc. He did not, however, recommend it for general use, from the increased number of figures required to express ordinary amounts.

most thorough and philosophical treatise on arithmetic in our language, and such a statement certainly deserves our most serious consideration.

The masterly and comprehensive report on the subject of weights and measures, made to Congress in 1821 by Mr. Adams, when Secretary of State, contains the following judgment : "The experience of France has proved that binary, ternary, duodecimal and sexagesimal divisions are as necessary to the practical use of weights and measures, as the decimal divisions are convenient for calculations resulting from them ; and that no plan for introducing the latter can dispense with the continued use of the former. * * * From the verdict of experience, therefore, it is doubtful whether the advantage to be obtained by any attempt to apply decimal arithmetic to weights and measures, would ever compensate for the increase of diversity which is the unavoidable consequence of change. Nature has no partialities for the number ten ; and the attempt to shackle her freedom with them will forever prove abortive."

So in the interesting paper of Dr. Ellis (in the *American Journal of Pharmacy*, Vol ii, page 202), the French decimal system is thus referred to : "Every one is struck, at the first glance of this system, with the beautiful simplicity which it derives from decimal arithmetic. It appears, however, to have been overlooked, that, although decimal arithmetic is admirably designed to facilitate the calculation of mere number, it is not equally well suited to the divisions of material things."

Much to the same effect has been the result of the commission appointed lately in England to consider the subject of a decimal coinage. The commissioners, after a full discussion and investigation of the subject, have very recently reported against any change ; their report being drawn up in the form of a series of twelve resolutions. The seventh resolution is as follows : "That as regards the comparative convenience of our present coinage, and of the pound and mill scheme, for the reckonings of the shop and the market, and for mental calculations generally, the superiority rests with the present system, in consequence, principally, of the more convenient divisibility of 4, 12, and 20, as compared with 10, and the facility for a successive division by 2 ; that is, for repeated halving, in correspondence with the natural and necessary tendency to this mode of subdividing all material things ; and with the prevalence of binary steps in the division of our weights and measures."

In the view, then, of this pervading law or principle of all human metrology, so well established, and so distinctly recognized, it becomes an obvious necessity, in adopting a decimal scale, to engraft upon it, the divisions of halves and quarters, at least (and in the case of the more commonly employed units, of eighths), if we would adapt it to the demands of the people, or if we would hope for its permanent establishment. It is true that this would involve a considerable number of subordinate divisions between one denomination of measure and the next below it, as it would be requisite to have separate and distinctive weights, for instance, for the unit (whatever it might be) for one and a quarter of

the unit ; for two, for two and a half, and for five ; and it is also true that the fractional values thus introduced would not be directly referable to the ordinary computations of decimal arithmetic—thus adding, somewhat, to the complexity and trouble of otherwise very simple calculations ; but this is a fault, not of the binary divisions themselves, but resulting from a radical and incurable defect in the decimal system. So long as we continue to count, to add, subtract, multiply, and divide by tens, so long must we submit to this inconvenience (undoubtedly a serious one) or we must choose the greater evil of abandoning all attempts at uniformity and consistency of system, and continue, as heretofore, to measure and to weigh by heterogeneous tables, while we perform the necessary operations of comparing, compounding, and distributing these values, by a method or ratio entirely dissimilar ; entailing upon ourselves the waste of time, labor, and patience, consequent upon a petty scheme of eternal and superfluous reductions.*

This horn of the dilemma is that which has been accepted by the coinage commission of England, to which a reference has just been made. The eleventh resolution of the Commissioners' Report is : "That the advantages in calculation and account-keeping, anticipated from a decimal coinage, may, to a great extent, be obtained without any disturbance of our present coinage, by a more extensive adoption of the practice now in use at the National Debt Office, and in the principal assurance offices, viz., of reducing money to decimals, performing the required calculations in decimals, and then restoring the result to the present notation." With our experience of a decimal coinage (notwithstanding its imperfections), this is not the horn likely to be selected by Americans in attempting a reform in weights and measures.

An expedient has been suggested by some, for facilitating division in decimal notation, which is ingenious, and deserves a notice. The project is to adopt a uniformly decimal system of weights and measures, but to estimate entirely by "cents"—by simply suppressing every alternate denomination ; thus, while reckoning decimally, we should traffic only centesimally. Our practical application of this method in all our money transactions, in which dimes are entirely suppressed in the market (though still having their place in the columns of the ledger) and our estimates made in *dollars* and *cents,* familiarizes our minds to the process, and enables us to see how such a system might be indefinitely extended, by the simple device of counting by double places of figures. The French table of weights would stand thus :

        100 deci-milligrammes make ....1 centigramme.

*"Perhaps it may be found by more protracted and multiplied experience, that this is the only 'uniformity' attainable by a system of weights and measures for universal use ; that the same material instruments shall be divisible decimally for calculations and accounts ; but in any other manner suited to convenience in the shops and markets ; that their appropriate legal denominations shall be used for computation, and the trivial names for actual weight or mensuration " (Adams's Report).

100 centigrammes	make ....1 gramme.
100 grammes	" ....1 hectogramme.
100 hectogrammes	" ....1 myriagramme.

This suppression of the alternate denominations would have the advantage of abolishing the very objectionable terms *decigramme* and *decagramme*. Instead of the extreme awkwardness of taking one quarter of a gramme ($2\frac{1}{2}$ decigrammes), we are furnished with the value in whole units, by taking twenty-five centigrammes, just as we say twenty-five cents instead of two and a half dimes.

Simple and taking as this proposal is, it is not free from serious objections. It, in fact, complicates rather than simplifies, by giving a very wide range for estimating values. While it thus multiplies the units, and enlarges the interval between them tenfold, it only furnishes us with a single additional bisection, namely, the quartering. An eighth would still require a fractional expression. Its benefit, therefore, bears no proportion to the increased trouble and confusion involved. The necessity universally felt for quaternal and octaval divisions, would infallibly operate here as it has in our currency ; and we should constantly hear of $37\frac{1}{2}$ hundredths of a pound ; $62\frac{1}{2}$ hundredths of a pint, etc., which would be, in no respect, better than $3\frac{3}{4}$ tenths, or $6\frac{1}{4}$ tenths. The truth is, we need more frequent denominations than decimal ones, rather than more distant stepping-stones ; and for some purposes, even the binary ratio of progression is not too slow. In looking over the various tables of weights and measures prevailing throughout Europe, it will be found that a large majority of the factors are 2, 4, and 8, with occasional resort to 3 and 6—the number 4 being, perhaps, the favorite number for the more customary denominations.*

Amid the conflicting claims of the numerous plans proposed for simplifying and uniting our incongruous metrology, there appears, at first sight, so much of irreconcilable contrariety, that it might be concluded that a combination of the respective advantages contemplated was hopeless and impossible ; and that we were only left to a choice of evils. A more careful scrutiny will however discover a philosophy in these very discrepancies, and furnish the elements of a practical concord. On the one side, the convenience of a system of divisions or multiples conforming exactly to that by which we are compelled to perform all arithmetical operations, is so obvious, and so universally recognized,† that the advocates of an entire decimalization are certainly justified in their zeal. On the other hand, the necessity of binal progression and division, though not so generally ack-

* This is rendered very apparent on turning over the pages of Woolhouse's little work on the " Weights and Measures of all Nations." No. 101, of Weale's Rudimentary Series.

† " The great improvement of having but one arithmetical scale for reckoning integers and fractions of every kind, * * * is one so obvious, and, withal, so little difficult, that it is a matter of surprise that it should not have been attempted till near a thousand years after decimal arithmetic was first introduced into Europe " (*Edinburgh Review* for January, 1807, Vol. ix, page 373).

nowledged, is by all who have given the subject a careful study, so fully appreciated, as being, at least, as fundamental as that of the decimal scale, that those who urge the retention of all such denominations as are mea- sured by the powers of 2, are no less justified. Which policy must, then, be sacrificed ?

"The elementary principle of decimal arithmetic," says Mr. Adams, "is supplied by nature to man within himself, in the number of his fin- gers. Whatever standard of linear measure he may assume in order to measure the surface or the solid, it will be natural to him to stop in the process of addition, when he has counted the tale equal to that of his fin- gers. * * * But while decimal arithmetic, thus for the purposes of computation, shoots spontaneously from the nature of man and of things, it is not equally adapted to the numeration, the multiplication, or the division of material substances either in his own person, or in external nature. The proportions of the human body, and of its members, are in other than decimal numbers. The first unit of measures for the use of the hand is the cubit, or extent from the tip of the elbow to the end of the middle finger ; the motives for choosing which are, that it presents more definite terminations at both ends, than any of the other superior limbs, and gives a measure easily handled and carried about the person. By doubling this measure, is given the *ell*, or arm, including the hand and half the width of the body, to the middle of the breast ; and by doubling that, the *fathom*, or extent from the extremity of one middle finger to that of the other, with extended arms—an exact equivalent to the stature of man, or extension from the crown of the head to the sole of the foot. For subdivisions, and smaller measures, the *span* is found equal to half the cubit, the *palm* to one-third of the span, and the *finger* to one-fourth of the palm. The *cubit* is thus, for the mensuration of matter, naturally divided into 24 equal parts, with subdivisions of which, 2, 3, and 4, are the factors ; while for the mensuration of distance, the foot will be found equal to one-fifth of the pace and one-sixth of the fathom " (*Adams's Report*).

"The fingers," says Dr. Lardner, "were naturally the first objects which presented to the mind the idea of number ; and they furnished, also, a set of natural counters by which the number of things might be marked and expressed. The fingers, being continually in view, familiar- ized the mind with the contemplation of every number of objects not exceeding ten. It was natural, therefore, that ten should be adopted as the number of objects to form the first group. * * * Although ten has been so generally adopted as the *radix* of systems of numera- tion, as to leave no doubt of its origin, yet it is not the only one which has been used, nor is it the only radix having a natural origin. The fingers of one hand rendered the number five familiar to the mind, before the conception of ten as a distinct number presented itself. It was even more natural and obvious, that the fingers should be contemplated as

two groups of five, than as a single group of ten" (*Treatise on Arith-
metic*, Book i, chap. i, p. 5-6).

The gradual and successive development of these scales, is so well set
forth in Mr. Peacock's valuable treatise, that perhaps no apology is neces-
sary for a somewhat lengthened extract from it, even at the cost of some
repetition.

"The decimal scale of numeration is not the only one which may be
properly characterized as a natural scale. In numbering with the fingers
we might, very naturally, pause at the completion of the fingers on one
hand ; and registering this result by a counter, or by any other means,
we might proceed over the fingers of the same hand again, or with the
fingers of the second hand, and register the result by another counter,
or replace the former by a new counter which should become the rep-
resentative of ten.   *   *   *   Again, the scale of numeration by
twenties has its foundation in nature, equally with the quinary and
denary scales. In a rude state of society, before the discovery of other
methods of numeration, men might avail themselves, for this purpose,
not merely of the fingers on the hands, but likewise of the toes of the
naked feet ; such a practice would naturally lead to the formation of a
vicenary scale of numeration, to which the denary, or the denary with
the quinary, or the quinary alone, might be subordinate.   *   *   *
Of other systems of numeration, the binary might be considered as
natural, from the use of the two hands in separating objects into pairs,
and from the prevalence of binary combinations in the members of the
human body ; but the scale of its superior units increases too slowly to
embrace within moderate limits the numbers which are required for the
ordinary wants of life, even in the infancy of society.   *   *   *
As the necessity of numeration is one of the earliest and most urgent of
those wants which are not essential to the support and protection of life,
we might naturally expect that the discovery of expedients for that pur-
pose should precede the epoch of civilization, and the full development
and fixing of language. That such has been the case, we shall find very
fully and clearly established, by an examination of the numerical words
of different languages ; for, without any exception which can be well
authenticated, they have been formed upon regular principles, having
reference to some one of those three systems which we have character-
ized as natural ; the quinary scale, whenever any traces of it appear,
being generally subordinate to the denary, and, in some cases, both the
quinary and denary scales being subordinate to the vicenary. In some
cases, also, we shall find, from an examination of primitive numerical
words conveying traces of obsolete methods of numeration, that the
quinary, and even the vicenary scales have been superseded altogether
by the denary" (*Encyclopedia Metropolitana*, art. " Arithmetic," Vol. i,
p. 371).

Decimal arithmetic thus appears to be coëval and coëxtensive with the
human race. It is, indeed, perhaps, the most universal of human insti-

tutions—at least as universal as language itself. From this universality, most writers have called it the "natural" system; but on examining the question whether the number *ten* possesses any intrinsic excellence or convenience to recommend it—any peculiar fitness as a ratio of geometrical progression, we find but one answer—it has none. It differs from any other number only in quantity, not in quality. So far from its presenting any merit or advantage over its compeers, it is almost the last number which a true science of arithmetic would have selected for the important function of a radix of numeration. Its universality flows simply from the fact that the necessities of man impelled a selection, in the very earliest infancy of the race, long before the invention of letters, and while yet a language was but slowly being formed; and the selection comes to us stamped with the crude impress of a most irrelevant accident. Had the six-fingered giant slain by Jonathan (2 Samuel xxi, 20) lived early enough to be the father of the first unreasoning tribes, we should have had a duodecimal arithmetic; or if, like the fowls of the air, we had usually but four toes to our extremities, we should now have been able to calculate only octavally; and in either event we should have been much more skillful computers than we are at present. *

Decimal numeration is "natural" then, only in the sense that *ignorance* is natural. The fingers have no more real or "natural" relation to the properties of number, than have any other organs or divisions of the human body; and mathematically or philosophically considered, the *digit* is, therefore, no more a typical *unit* than a tooth (of which there are thirty-two), or the leg of a spider (of which there are eight), or the petal of a flower (of which there may be any number). Nor have any but the most ignorant races—those without a literature and an alphabet— ever occasion to group and tally by their fingers. Only from unlettered savages could such a scale, therefore, have deen derived.

It has been a favorite theory with a certain class of thinkers that primitive man was a highly civilized being—"a scholar and a gentleman;" and that the decay of states, and the decline of civilizations so unfortunately frequent in his history, but manifest his prevailing tendency to degeneration. Our universal arithmetic furnishes us with one of the most striking refutations of such a fancy. Wherever over the broad earth, the decimal scale exists, there have we the enduring monument of the ancestral savage—counting by his fingers or his naked toes.†

* "There can be no doubt that if man had been a twelve-fingered animal, we should now possess a more perfect system of numeration than we do. Whatever be the radix of the scale, it would always be a convenience to be able to subdivide it with facility, without resorting to the more refined expedient of fractional language" (*Lardner's Arithmetic*, chap. i, p. 21).

† The German word for ten—*zehen*—signifies "toes," being the plural of the word, *zehe*, We do not generally or readily recognize this intellectual association in our own language; and yet the Saxon word—*ta*—a "toe," is in the plural *tan*. The *daktul* $(\delta \alpha \kappa \tau \nu \lambda \nu \varsigma)$ of the Greeks, and the *digit* (digitus) of the Romans, which signified either "finger" or "toe," appear evidently affiliated to the *deka* $(\delta \varepsilon \kappa \alpha)$ of the one and the *decem* of the

Had any intelligent forethought ever presided over the inception of a numerical scale—had any comprehensive conception of the uses and purposes of figures, in any single instance guided the selection of a ratio for their multiplication—that ratio must inevitably have been something else than *ten ;* the duplication of an odd number—incapable of any other division—neither a square, a cube, nor other power of any integer—and in its successions among the most inefficient for the expression of fractional values, or for the extraction of roots. And if among the patriarchs of the human family, a rational scale had ever been so devised, some traces of this wiser system must have been found, to give a "sign" and memento of man's pristine elevation.

"The number ten," remarks Mr. Anderson, in his treatise on Arithmetic, "has been adopted by every civilized nation for the radix of the numerical scale. It has no peculiar advantages to recommend it, and seems to have been selected for that important function, merely because it expresses the number of the human fingers. We must regret that a circumstance so totally unconnected with every scientific consideration, should have determined an elemental principle, of the last importance to one of the most abstract, as well as one of the most useful of all the sciences ; and that the decimal notation should still be retained, notwithstanding its evident imperfections, and the superior claims of other scales" (*Edinburgh Encyclopedia ;* edited by Sir David Brewster, art. "Arithmetic," Vol. ii, page 411).

An able and philosophical writer in the *Edinburgh Review* holds very similar language. "Ten has indeed," he observes, "no advantage as the radix of numerical computation ; and has been raised to the dignity which it now holds, merely by the circumstance of its expressing the number of a man's fingers. They who regard science as the creature of pure reason, must feel somewhat indignant that a consideration so foreign and mechanical, should have determined the form and order of one of the most intellectual and abstract of all the sciences" (*Edinburgh Review,* for January, 1807, Vol. ix, page 376).

A large number (perhaps even a large majority) of the well-educated have been accustomed to regard the decimal system as possessing a peculiar beauty and expressiveness, from the great facility with which the ordinary operations of arithmetic are performed by it. Indeed, after laboring at the tedious and troublesome reductions of compound num-

other ; although the genealogy (as in English) was probably more ancient than the languages themselves. So uniform are the laws of mind and matter, that we have only to select some rude and isolated tribe of modern savages to discover with a naturalist's confidence, the exact process of development in numeration, with the aborigines of our race, milleniums on milleniums ago. Klaproth, in speaking of the inhabitants of the peninsula of Kamtschatka, says: "It is very amusing to see them attempt to reckon above ten ; for having reckoned the fingers of both hands, they clasp them together, which signifies ten ; they then begin at their toes and count to twenty ; after which they are quite confounded, and cry " Matcha," that is, where shall I take more ?" (*Sprachatlas,* page 16.)

bers (consequent upon other scales of progression) unfortunately so often required to be made, the relief of a simple addition or multiplication in the homogeneous units of our common scale, is too striking not to excite a feeling of admiration for the easier process. It appears not to be gene- rally considered, however, that this facility of computation is in no respect due to the series of "tens" by which we count, but is derived exclusively from the admirable notation in which the series has been clothed, and through which alone, we are in modern times made acquainted with it ; and from the perfect conformity of the notation to the series. Any other scale will be found to exhibit an equal facility, if the same notation be employed, and made to correspond strictly with the selected scale. If, like the old Arabian philosophers, or like the ancient Greeks and Romans, we were compelled to calculate by a set of *alphabetic* numerals, we should be able to better realize how much we are indebted to that simple and yet grand invention of India, the "cypher figures," or the set of figures with the device of local value.* This system of numerical lan- guage presents us with a formula of geometrical progressions, so illimit- able in range, and yet so perfect in its conciseness and distinctness, that it transcends all conception that the ingenuity of man in all coming time shall ever be able to improve it.

Though from a remote antiquity familiar to the Hindoos (that wonder- ful people from whom the civilized world has derived so much), it was wholly unknown to the nations of the earth until comparatively modern times ; having been first introduced into Arabia, less than a thousand years ago, and from thence by slow and successive centuries into the various languages of Europe.

However much the Arabian philosopher to whom belongs the honor of having first transplanted the Sanscrit Arithmetic into his own country, may have been impressed with its great power and beauty, he could hardly have appreciated, to its full extent, the importance and magnitude of the gift he was instrumental in presenting to the civilized world ; a transfer which Sir John Bowring in his "Decimal System" (chap. ii, p. 22) has characterized as "the greatest step ever made towards the introduction of a universal language among the nations of the world." The Hindoo numerals, from the channel of their introduction into Europe, were gen- erally called the "Arabic figures"—a title they still commonly retain, though it is one hardly just to the people with whom these figures had their origin.

Now although this Hindoo notation has never been popularly applied to any other than the decimal scale, it is obviously a formula of universal applicability ; and if made use of to express a system of figures with any other radix than *ten*, would give the same facility to all calculations per- formed by that system.

Abstracting, for a moment, all specific value from the terms "units," "tens," "hundreds," and "thousands," and regarding them merely as

* See note A, page 357.

symbols of *local value* (designating only the orders of units), we may exhibit in a tabular form, a series of scales, with the successive increments of value for each place, according to the radix, or ratio of geometrical progression selected. In the following table the letter "U" in the top line denotes a "unit;" that is, any figure which may occupy a single place:

Table of Arithmetical Scales.

Hindoo Notat'n	U.	UO.	UOO.	UOOO.			
Scale.	Units.	Tens.	Hundreds	Thousands.	Radix	Logar'm	Places
Binary.	1	2—	4—	8—	2	.301	$\frac{10}{3}$
Ternary.	1—2	3—	9—	27—	3	.477	$\frac{25}{12}$
Quaternary	1—3	4—	16—	64—	4	.602	$\frac{5}{3}$
Quinary	1—4	5—	25—	125—	5	.699	$\frac{10}{7}$
Senary.	1—5	6—	36—	216—	6	.778	$\frac{9}{7}$
Septenary.	1—6	7—	49—	343—	7	.845	$\frac{20}{17}$
Octonary.	1—7	8—	64—	512—	8	.903	$\frac{10}{9}$
Novenary.	1—8	9—	81—	729—	9	.954	$\frac{20}{19}$
Denary	1—9	10—	100—	1000—	10	1.000	1
Duodenary.	1—11	12—	144—	1728—	12	1.079	$\frac{25}{27}$
Quaterdenary. . .	1—13	14—	196—	2744—	14	1.146	$\frac{50}{57}$
Senidenary.	1—15	16—	256—	4096—	16	1.204	$\frac{5}{6}$
Octonidenary. . .	1—17	18—	324—	5832—	18	1.255	$\frac{4}{5}$
Vicenary.	1—19	20—	400—	8000—	20	1.301	$\frac{10}{13}$
Tricenary	1—29	30—	900—	27000—	30	1.477	$\frac{25}{37}$
Quadragenary . .	1—39	40—	1600—	64000—	40	1.602	$\frac{5}{8}$
Quinquagenary.	1—49	50—	2500—	125000—	50	1.699	$\frac{10}{17}$
Sexagenary	1—59	60—	3600—	216000—	60	1.778	$\frac{50}{89}$

The number of places for each scale is inversely as the logarithm of the radix.

The most striking feature displayed by such a comparison of the different scales is the rapid increase of value in the higher ratios, as compared with the lower. While the *ternary* scale, for example, requires four figures to express so small a number as 27, the tricenary scale expresses one thousand times as many, by the use of no more places. The very first inquiry would, therefore, naturally be (in the absence of any other consideration), which would be found more convenient—a very small radix, or a very large one.

The first and lowest scale of the series—the *binary*—presents, with some disadvantages, many very remarkable advantages. In the first place it requires but a single figure, 1 (together with the cipher for determining its place), to express with facility and precision all the values within the reach of figures.* According to the law of the Hindoo notation, by

* It was in reference to this curious property of the scale, that a medal struck in honor of Leibnitz, and to commemorate his invention of the binary system, bore on its reverse, the striking inscription: "Omnibus ex nihilo ducendis sufficit Unum." Unity being very commonly regarded as the symbol of the Deity (*Peacock's Arithmetic*, Encyclopedia Metropolitana, Vol. ii, page 392).

which every zero multiplies all the value that precedes it, by the amount
of the radix, it results that the addition of a cipher to the figure 1, would
of course multiply it by two (instead of by ten as in our common sys-
tem)—the addition of two ciphers, by two times two, or four (instead of
by a hundred)—the addition of 3 ciphers, by eight ; of 4 ciphers, by six-
teen ; of 5 ciphers, by thirty-two, etc. The first fifteen numbers would
read thus : 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101,
1110, 1111.* The present year, 1887, would require eleven places of fig-
ures to express it; namely, 11101011111. Fifty places of figures (or 1
and 49 ciphers) in the binary system, would require but fifteen places of
figures in the decimal system. One hundred places of the binary (1 and
99 ciphers) would require thirty places of the decimal. So that the for-
mer system would involve, on an average, the constant employment of
about three and a third times more figures in all our arithmetical opera-
tions, than the latter system, or that in common use. This increased
expenditure of time and manual labor would evidently be a very serious
inconvenience. On the other hand it must be considered that the writing
down of any given mass of figures, in only two characters (always either
1, or a cipher), would be much more easy and expeditious than if the
mass consisted of ten different characters ; so that the actual increase of
trouble should be set down at probably not more than *double* that we
have at present. This much quantitatively. But in the quality of the
work done, the difference will be found immensely in favor of the binary
scheme. In the first place no tables would be required to be committed
to, and retained by, the memory ; either of addition, of subtraction, of
division or of multiplication ; not even the fundamental "twice two make
four." Every form of calculation would be resolved into simple numera-
tion and notation. In fact, calculation as an effort of mathematical
thought, might be said to be entirely dispensed with, and the labor of
the brain to be all transferred to the eye and the hand. A perfect
familiarity with the notation of the scale, and with the simple rules of
position, would enable the operator to determine in every case by mere
inspection whether the next figure should be a 1, or an 0. It follows
that the only errors possible in such a work would be the merely clerical
ones of the eye or hand ; and when we reflect that a large majority of the
arithmetical errors committed are usually those of the brain, fatigued or
bewildered by the constant strain upon the attention and memory, this
consideration of the increased accuracy of such a system is one of the
very first importance in estimating its value. To many, the relief it
proffers in exchanging head-work for hand-work will appear no trifling
recommendation ; and it may well be doubted, whether in all important
and lengthy calculations, the binary system would not be found to afford
a real economy of labor, instead of an increase as has been generally
supposed.

It has been previously noticed, that the great Leibnitz, the rival of
* See note B, page 359.

Newton in the invention of the "Differential Calculus," proposed this system and zealously urged its adoption; although he thought that for more common purposes it would be found too prolix. "De Lagny took the trouble of constructing logarithms on the principles of this arith- metic, as being more natural than those usually employed. * * * He even proposed to substitute binary arithmetic for logarithms, affirm- ing that it was more simple and expeditious, and conducted to the object in view in a less indirect manner" (*Anderson's Article on Arithmetic, in Brewster's Edinburgh Encyclopedia,* Vol. ii, pp. 376 and 409). The same writer adds that "Dangicourt has applied the binary notation with greater success to progressions, and proved that the laws of a series may be detected by it more easily than by any other scale." This results, it may be as well to state, from the fact that "circulating periods" of figures return far more frequently in this scale than in any other.

The *Ternary* scale, although it is also a very simple scale, has nothing else to recommend it; being incapable of integral bisection, and having very nearly the redundancy of the binary scale, without one of its advan- tages. It may be regarded as one of the most objectionable of all the scales; and indeed none of the odd numbers could, for a moment, be accepted as a suitable radix of notation.

The *Quaternary* scale, as derived from the second power of the binary scale, has many of its excellences. While it employs less than half the number of digits, of the common or denary scale, to task the memory and attention, it requires only about five places of figures, for three of the latter. It combines, therefore, great simplicity of structure, with a mode- rate range of notation, and would form a very convenient and practicable system of numeration; while it would furnish an admirable scale of division for weights and measures of all kinds. It is said by Balbi, that a very low and ignorant tribe of Indians in South America—the Guranos— had names for only four digits, and that after counting these a second time (to eight) they were unable to proceed any further. The correctness of this account appears, however, to be exceedingly doubtful. It is remark- able, too, that Aristotle mentions a tribe of Thrace as being unable to count beyond four—a statement equally incredible.

The *Quinary* scale, whose notation would require ten places for seven of the *denary,* has nothing to recommend it; and yet from the accident of man being afflicted with five fingers, it has generally formed the basis of the scale in common use, and traces of it are to be found in perhaps a majority of the nations of the earth. The numerals of Malay and Java were anciently, for the most part, quinary, in subordination to the vice- nary grouping. A trace of this system is also seen among the ancient Greeks, in their word πεμπαζεσθαι (to count by fives); as it is among the Romans in their notation of numbers above 5, 15, etc. The Persian term for "five" is *pendja;* and *pentcha* signifies the expanded hand. Among the South Sea Islanders, the inhabitants of New Caledonia and the Hebri- des, as well as the barbarous tribes of Northeastern Asia, the quinary

scale appears still to prevail. The central tribes of North America show also traces of this digital period ; and they are frequent among the innumerable languages of Africa. Thus with the Jalloffs, the word for "five"—*juorom*—signifies the "hand." So with the Foulahs, the Jallonkas, the Fellups, etc. There are no examples, however, of the number five ever having been used as a true radix of notation ; that is, as a direct ratio of continued progression ; $5 - 5 \times 5$ (25) ; $- 5 \times 5 \times 5$ (125), etc. The quinary scale has seldom gone further than 20.

The *Senary* scale would require about seventeen places of figures for thirteen of the common scale ; and its notation would therefore have about a one-third greater extent. Though not one of the most desirable scales, it would be much superior to the denary system. The simplification arising from the reduction of its digits, would much more than counterbalance the extension consequent on the increase of its places. Like the denary scale, it admits of but one bisection ; but it possesses the great superiority of admitting at the same time of a trisection. No examples of this scale are to be met with ; although it is said to have been at one time decreed in China, by the caprice of an Emperor, who had conceived some astrologic fancy for the number six.

The *Octonary* scale approaches very nearly to the common scale in its capability of expression, as it requires on an average but one-ninth more places of figures to represent any given amount ; that is, ten places of this scale would be equivalent to nine places of the denary. Being derived from the third power of the binary scale, it possesses most of the advantages of that system ; though not its admirable simplicity. Like the quaternary, it admits of continued bisection down to unity ; and, of course, of indefinite bisection below 1, by the simple expedient of an inverted, or negative notation (as in decimal fractions). As a perfect cube, it has peculiar advantages both as a radix of numeration, and as a ratio of progression or of division for weights and measures ; and in the latter respect particularly, there is, perhaps, no other number that would so well express the average range of a convenient metrical multiple.

The *Denary* scale* may be said to present a tolerably convenient mean between the prolixity of a very small radix, and the intricacy of a very large one ; besides which, it possesses the immense advantage of a universal establishment. But beyond this, there is nothing to be said in its behalf. Intrinsically, it is one of the most imperfect and troublesome scales which could be selected. Still, the inconveniences of the system should be very serious and very apparent, and the claims of any rival scheme very unquestionable, to justify the advocacy of a change, which

* The name "Decimal," by which our present system of arithmetic is commonly designated, appears not to have a perfect propriety. The terms "Octaval," "Nonal," "Decimal," "Duodecimal," etc., are derived from the Roman "*ordinals*," and belong to the series Primal, Secundal, Tertial, Quartal, etc. The idea really involved is not that of relation to a *tenth*, but of a relation to a grouping *by tens*, and would require the term "denal" or "denary"—from the Roman "distributive" numerals, of which the terms "binary," "ternary," etc., commence the series.

would root up all our established forms and habits of calculation—which
would destroy the accumulated products of centuries of industrious
thought and toil—which would entail upon us generations of new labor
to attain even the same standard of tabular detail, and statistical informa-
tion, now possessed ; and which, more than all, would wholly demolish,
and perhaps hopelessly, that uniformity so essential to the language of sci-
entific investigation, and so universally conceded to be one of the most
important aims and results of every project of metrical or numerical im-
provement.

Upon this basis must the question of so radical a revolution rest.   But
if it is shown that uniformity in many other relations than those of simple
number, and no less vital to the interests and welfare of the race than this
boasted uniformity of figures, has constantly and irretrievably been sacri-
ficed to this great idol—if it is established by the voice of all experience
that neither national nor international standards of length, of weight, of
area, of volume, or of value, of any single subject, in short, to which
these figures can be usefully applied, have ever the slightest hope of ob-
taining a general authority under the dynasty of this "universal" power
—then must it be dethroned, for very uniformity's sake, and a new dis-
pensation introduced, developed from such principles, and invested with
such attributes, that it may rationally be expected to gain at length a uni-
versal ascendancy, through the concurrent approval and adherence of all
intelligent nations.   For the attainment of a real uniformity, there seems
no other process or alternative ; and for such an attainment, no sacrifice
of temporary convenience could be held to be too great.   The faults of
the denary system are too radical to be amended—too obnoxious to be en-
dured.   Sheltered by the inertia and conservatism of inveterate habit, it
has been tolerated already much too long.   The unskillful contrivance of
an early age, it is all unsuited to the wants or uses of an adult manhood
of the race.

The *Duodenary* scale has over the denary the advantage of allowing two
bisections, and, at the same time, like the senary scale, of admitting of a
trisection.   Its variety of factors, 2, 3, 4, and 6, give it a much greater
power of expressing fractional values than any scale below it, or imme-
diately above it ; and it has accordingly been always found a conve-
nient and favorite number for metrical divisions.   The acres, the feet, and
the pounds of the Romans were all divided by 12 ; as are the foot, and
the Troy pound, still with us.   The signs of the Zodiac, the months of
the year, and the hours of the day, have illustrated the number from the
remotest antiquity.   In the old French measures of length, not only the
foot was divided into 12 inches, but the inch into 12 lines, and the line
into 12 points.   The "dozen," the "gross" or (12^2) and even the "great
gross" (or 12^3) are widely used in trade at the present day for the
package of a variety of articles.   From the many acknowledged advan-
tages of the duodenary scale, it has found frequent and warm advo-
cates for its adoption as a system of numeration.   In the necessity of

two additional integers, it would offer however a considerable increase of complexity and mental labor ; while the economy of places in notation could scarcely be regarded as appreciable—25 of the duodenary being required for 27 of the denary. As compared with the octonary, it would require 5 places, where the latter would require 6 ; so that while its digits are more by fifty per cent, the excess of the other in places is only twenty per cent. But there are far more important considerations than these, which unfortunately oppose themselves to the adoption of this system, as the best substitute for the denary, notwithstanding its admitted features of superiority.

The most fatal objection to the radix 12, is that it permits only a single bisection beyond that given by the radix 10. The quality of continued divisibility, we regard as paramount to all others ; not merely for the convenience of art and trade, universal as their requirements are, but even for many scientific purposes ; and however valuable the property of a *varied* subdivision (as that furnished by the duodenary scale), experience has fully demonstrated, what is clearly seen by theory, that no aliquot parts can ever be as widely useful as the binal fractions. Another objection to the 12 scale, somewhat allied to this, is that the number is not a *power* of any integer—a point, as we shall discover, of no slight importance. In this respect, it may be remarked, the number nine (awkward and inconvenient as it undoubtedly would be as the basis of an arithmetic) would have several advantages over the number ten, and even over the number twelve. A third objection to the scale under consideration, which, though not so striking, is yet no less real : the radix is too large. On the simple score of size, there must be somewhere in the indefinite range of scales, a point where we should expect to find the most convenient medium between the inexpediences of opposing difficulties ; and although this most advisable limit of magnitude may not admit of very precise determination, the question is one of too great consequence in the comparisons we are making, not to deserve a special attention.

The *Seni-denary* scale presents many excellent points, the number 16 being both a square, and a fourth power, and admitting of indefinite division by two. Its only disadvantage is the incommodious number of digits it would require ; while its notation would yet economize only a single place of figures in every six places required by the denary scale.

The *Vicenary* scale furnishes no single point of merit which could recommend it to our acceptance, unless its divisibility by four should be regarded as giving it a superiority to the denary. With an exceedingly troublesome and unwieldy range of digits, it would reduce the extent of our common notation only from 13 to 10 places. Man was, however, unfortunately born with 20 extremities, or branches to his limbs, and hence traces of what may be designated a rudimentary vicenary scale, are to be met with among many nations, both ancient and modern. In ancient Phœnicia and Palmyra, the system of numbering by twenties, as far as

the hundred, prevailed ; and from these nations it was derived by the
Celtic languages, in all of which its remains are still found. Among the
Scandinavians, also, is found a vicenary numeration. The Greenlanders,
having counted fingers and toes in periods of five, designated the num-
ber 20 by the word *innuk*, which signifies a " man." If they have occa-
sion to proceed higher, the expression for 40 is *innuk arlak*—" two men"—
etc. A similar method existed among the Aztecs of ancient Mexico ; as
well as among the tribes of South America. The Teutonic races retain
in their languages the traces of the ancient "score," and in parts of
England, counting by scores, or twenties, is still quite usual. The trans-
lators of our Bible have frequently (though by no means uniformly) in-
troduced this mode of enumeration. Thus we have "three-score and
ten" (Ps. xc, 10)—"three-score and twelve" (Numb. xxxi, 38)—"three-
score and fifteen" (Acts vii, 14)—"three-score and seventeen" (Judges
viii, 14), etc., etc. The mode of numbering still in common use in
France also exhibits a very remarkable retention of the antiquated vice-
nary system.*

This scale is not, as might be supposed, an extension of, and attempted
improvement upon, the decimal system. On the contrary, it almost uni-
versally preceded it ; and its employment belongs to the very earliest and
rudest stage of barbarian society. It betrays a period of human intelli-
gence, so destitute of all resource, that fingers and toes must all be pressed
into service, to meet the common wants of number ; and when these have
been exhausted, there has been found among some tribes, no power of
thought or word or symbol for aught beyond. It indicates a period long
before a conception of any expedient for numerical expressions had
dawned upon the savage brain ; and hence there is no example of the
vicenary scale having ever been extended even as far as to the second
place of figures, or to 20 times 20 ; nor probably even beyond one hun-
dred. It is evident that when the necessity for expressing larger num-
bers began to be felt, the cumbrous scale of added "toes," must soon be
dropped, and the range restricted to the more manageable mechanism of
the ten "fingers." And, accordingly, we find the imperfect vicenary to
be always overlaid by the denary, with glimpses of the former still appear-
ing through its supplanter.

The *Sexagenary* scale deserves notice only from its historical interest in
having been from a very remote antiquity employed for particular pur-
poses among the people from whom we derive our arithmetical notation—

* "The French nomenclature is for the most part purely decimal. The decimal sys-
tem is observed from twenty (*vingt*) to sixty (*soixante*); here we find a vestige of an old
vicenary scale. Seventy, instead of being *septante*, as the decimal system would require,
is *soixante-dix* (sixty-ten); seventy one, *soixante-onze*, (sixty-eleven); seventy-two, *soixante-
douze* (sixty-twelve), etc. Eighty, instead of being *octante*, is *quatre-vingt*, or four twen-
ties, and ninety is *quatre-vingt-dix* (four twenties ten); ninety-one, *quatre-vingt-onze* (four
twenties eleven), etc. Thus twenty becomes the radix of the system from sixty to a
hundred." (*Lardner's Arithmetic*, page 11.)

an employment which has been perpetuated throughout Europe and America, to the present, in the smaller divisions of time and of the circle. This scale is of course far too cumbrous in the range of its units to have ever had a true notation, or to be ever possible as an actual system, founded on its own radix. With its enormous complication of figures, it would still require about $\frac{9}{16}$ (or more than half) of the places of the common system to express its values. It has been found very useful, however, in its limited application, both from the rapidity of its progressions, and from the remarkably varied range of divisibility it permits. The number 60 is divisible by 2, by 3, by 4, by 5, by 6, by 10, by 12, by 15, by 20, and by 30 ; and has indeed the greatest number of aliquot parts of any number below 96.

Our " minutes " and " seconds " of the degree and of the hour have thus an Oriental origin. In India, however, from whence the scale was derived, these divisions, as applied to time, had not the same value as with us ; as there the day itself was divided into 60 parts, called *guries* (hours of 24 minutes), each *gurie* into 60 parts, called *polls* (minutes of 24 of our seconds), and lastly each *poll* into 60 *mimiks*, or twinklings of an eye (four-tenths of a second). It is believed that this division of time is re- tained by the Hindoos to the present day. They also employ a period of 60 years, as we do the century.

In its astronomical application this scale has been found exceedingly useful. The properties of the circle require that it should frequently be divided into sixths, as well as into quarters ; the sixth being, as is well known, the radial arc, or that whose chord is exactly equal to radius. The zodiacal or ecliptic circle of the heavens had, from the earliest antiquity, been divided into twelfths, a period representing approximately the move- ment of the sun during one lunation. As this comprised very nearly 30 days, the " sign " became naturally divided into 30 degrees ; and this ex- presses so closely the arc of the earth's orbit described in one mean solar day, that when the earth is moving slowest (or at its aphelion), it falls but three minutes within one degree, and when it is moving fastest (or at its perihelion), it exceeds the degree by only a single minute. The radial arc of two " signs," or 60 degrees, suggested its own subdivisions. Hence was derived the table of 60 seconds to the minute, 60 minutes to the de- gree, and 60 degrees to the sextant—6 of these completing the circle. This system, answering so well the requirements of various division, was introduced from India into the Alexandrian school by the illustrious Ptolemy,* who did so much toward giving astronomy a scientific form. The sexagenary scale has never, however, been computed by any other than a denary radix. It must excite surprise, therefore, that the Hindoo notation of the scale was not also introduced by Ptolemy at the same

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* Although the sexagesimal arithmetic is commonly ascribed to Ptolemy, it is probably an Eastern invention. The Indians, to this day, employ the sexagesimal division of time" (*Edinburgh Encyclopedia*, art. " Arithmetic ").

time ; and the world thus put in possession of this grand invention eight centuries earlier than it was by the Arabic importation.*

In our survey of the principal scales, from which alone a selection could be made for popular uses, we have found that there are certain in-cidental, but opposing advantages, incompatible with each other ; and that no scale, therefore, could possibly furnish a maximum of every con-dition that might be thought desirable. Thus the binary scale affords so admirable a simplicity, beauty and facility, that it would have to be re-garded the perfect system, if its redundant employment of figures (the necessary consequence of its simplicity), did not render it unsuited to the small and constant calculations required in the daily course of trade. On the other hand the manifold divisions permitted by the sexagenary scale give it convenient qualities, impossible to the lower scales ; but here we find a complication so onerous that it would appall the most inveterate of calculating monomaniacs.

The conditions, however, that are really most essential to an arithmeti-cal radix, are so few and precise, and their requirements so imperative, that there is little difficulty in deciding upon "the best possible scale of numeration." The first consideration would naturally have regard to the size of the radix, in order to assign certain limits within which our scale is to be found. To realize a maximum convenience, it must be neither too large, nor too small. We have seen that while the notation of places (and the consequent labor of transcription) diminishes very slowly with the ascending scales—the tax upon the mental faculties increases in a far more rapid ratio. The labor of mere *calculation*, which may be estimated at zero for the binary scale, advances materially, and in a compound ratio with every figure added to the radix. Were we then required to choose between any two scales—separated by a considerable interval, that is, be-tween a very small one and a very large one (no other insuperable objec-tion being supposed), we should adopt, unhesitatingly, the smaller one. The advantage imagined by some, of the great expressiveness of a rapid increase of value, is wholly illusory. It needs comparatively very few figures, in any case, to carry us not only beyond all true conceptions of

*The Greeks, like the Hebrews, Arabs, and all other nations excepting the Hindoos, employed an alphabetic numeral ; and it is a somewhat curious circumstance that our modern character for the cipher was derived not from India or Arabia, but from Alex-andria. The Hindoos indicated the cipher place by a simple dot (.), and the Arabians, in borrowing their system, did the same; until the sexagenary system, introduced by Ptolemy so many centuries before, supplied them with a new character. This philosopher, find-ing a frequent occasion to mark the absence of a particular denomination (as "no min-utes," or "no seconds"), in order to avoid mistake employed the first vacant letter of the alphabet for that purpose. As the Greek numeral for 60 is the letter $\bar{\varsigma}$, all those which followed would be useless for the sexagenary scale ; hence the next letter, o (omicron), naturally became the empty counter. This notation became established by long habit among the astronomers of Alexandria, Constantinople, and Arabia ; and finally crept into the Hindoo system of numerals. Thus to the accidental position of the Greek letter *omicron*, which happened to represent *seventy*, we are indebted for the present form of our modern cipher as a circle, instead of a decimal period.

magnitude, but beyond all rational requirement of any real calculations we can devise. There is, in the law of continued geometrical progression, even on its lowest scale, a power so overwhelming, that we feel we have no extra wonder or admiration left to spare, upon these "infinites of higher order," and confess to a predilection not to travel at such dizzying speed.

The world has had some centuries of experience in the denary arithmetic. We are all familiar with the laborious and tedious discipline by which its practice is acquired ; and we are all conscious of the exertion of thought demanded to perform a lengthy operation in figures. When we consider the amount of time bestowed in training youth in this branch of learning (and yet the fact that not one-half so trained are really expert in calculation), we must record it as our deliberate conviction, that *the denal radix is too large*. We believe that a lower figure would give the true desideratum—the *minimum* of labor. Nay, as between the scale of ten and that of six, we incline to the opinion that the latter would be found the more convenient notation. Its labor, both of acquisition and of exercise, would certainly be far less than half, while its figures in use would be only about a third more. *A priori*, we might expect that a scale established in rude and inexperienced times (were it not that it was really determined by an arbitrary and extraneous circumstance) would be too large in its ratio of progression—rather than too small ; and that a more enlightened age would find it convenient to reduce it ; just as we have seen to occur with the vicenary and the denary scales, in their early history.

The second essential that should be demanded in a radix is that it must admit of indefinite bisection, or, in other words, that it must be found among the powers of two ; namely, 4, 8, or 16. As 4 is probably too small, and 16 certainly too large, we have the octonary scale alone left to satisfy our most vital two conditions of a medium size, and a complete divisibility. The concurrence of these qualities in any one scale, and in that one alone, is sufficient to establish its claims against all competitors. There is but one scale which could have any pretensions to be considered a rival, or which would be likely to find intelligent advocates ; and that is the duodenary. Much stress has been laid upon the number of its aliquot parts. That this quality is a highly useful one, we frankly acknowledge, but yet, as we maintain, not nearly so useful as that other quality this radix lacks, the facility of successful halving. The number 12 is not a power ; the number 8 is a cube ; an important advantage in several respects, but particularly in the application of this scale to a system of metrology, from the simple relations thereby established between the measures of length and those of volume—by which both weights and measures of capacity are determined. All that has been said on the subject of the denary being too large a scale, applies with much greater force against the duodenary. And, finally, we believe that a large majority of the mathematicians would give their vote unhesitatingly in favor of the octonary arithmetic. It appears to combine advantages of

the very first importance, and those impossible in any other scale. While perfectly adapted to the highest requirements of science, it is as exactly suited to the trivial wants and petty occasions of our daily life. It possesses a degree of simplicity the most attainable without a sensible increase of figuring. The simple suppression of the largest two digits of our common system (8 and 9) throughout every place of figures, would be found to reduce the working labor by at least one-half. In choosing between a radix of a second power (as 4), and one of a third power (as 8), the latter would for several reasons be preferred. It would undoubtedly be advantageous for it to be at the same time both a square and a cube. But unfortunately we can meet with no such favored number, until we reach the period 64. Our octonary radix is, therefore, beyond all comparison the "*best possible one*" for an arithmetical system.

After this somewhat tedious preparatory exposition, we now propose to briefly develop the scale of numeration thus selected ; and to derive from it an ideal system of measures, based throughout upon the leading ideas of the French system ; availing ourselves, as we believe, of every beauty and refinement offered by it, and avoiding every difficulty and defect inherent in it. Let us attempt to employ our proposed scale of number in the first place, by putting it in an intelligible form. Although we might readily discriminate between the octonary and the denary notation by the simple expedient of using a somewhat different type, of our common figures (suppressing the 8 and the 9), yet even with this device, the association of local value is so strong that it would not be easy to avoid confusion of idea in attempting to read and understand the unfamiliar conversion. It will be found much easier, therefore, to devise a set of characters for the octonary scale ; which should be entirely distinct both from the letters of the alphabet, and from our ordinary figures. To assist us still more in reading them, these characters might be made significant symbols, by the number of lines employed in the construction of each, though this would be a matter of very little importance in a form of character that should be permanently adopted. The characters should all be simple ; they should all have the same size, for the obvious convenience of typographic "dress ;" and they should be so distinctive, that no one could easily be mistaken for another. Let us then represent *one* by ᒪ; *two* by ᒥ; *three* by ᒣ; *four* by Ϝ; *five* by P; *six* by ᕒ; and *seven* by ᕼ; the cipher having no intrinsic value, may very well continue to be still represented by ᑎ. Our eight *digits*, then (if we must still use so barbarian and unmathematical a designation),* would stand thus : ᑎᒪᒥᒣϜPᕒᕼ.

In reading these octonary numbers, a distinctive name for each, as

* It has been sometimes remarked by advocates of the octonary arithmetic, that if our stupid ancestors had only used their thumbs as the counters of the digits, they would have found that they had but eight fingers, and we should then have had the octaval period—"founded in nature." It may be supposed from the preceding discussion of this subject, that we attach but little importance to such a consideration.

well as for the places occupied by them, would become even more necessary than a distinctive form. The terms "ten," "hundred," "thousand," especially, are too essentially decimal in their origin, and too ineffaceably stamped by usage in their significance, to permit their use in any novel application. The names, like the symbols, should be both as simple and as distinct as possible. The simplest name is a monosyllable, containing but one consonant and one vowel sound. Let this then be the rule of our numerical vocabulary. It will be convenient and even advisable to preserve a resemblance to the popular numerical language, that the analogy of structure may be the more apparent. The word one will naturally give us the French "un ;" two will give us "du ;" three will give us "the ;" the consonant sound being really a simple one, although requiring two letters in our language. The word "tre" would have been better, as being very near the Latin tres, the Greek treis and the original Sanscrit tri ;* but the double consonant excludes it under our rule.

* It is a matter of curious philological interest to trace the Sanscrit or ancient Indian parentage of all our modern European languages, especially in the names of the numerals. In this particular the different vocabularies of the numerous and wide-spread races,—of the Celtic, the Romaic, the Sclavonic and the Gothic, with its two great families of the Scandinavian and the Teutonic, appear only as dialects of each other. The names of the first ten numbers, in a few languages, are here selected, mainly from the Introduction to Bosworth's Anglo-Saxon Dictionary :

Sanscrit.	Persian.	Greek.	Roman.	Welsh.	Gothic.	German.	Saxon.	English.
Aika	yika	eis, en	unus	un	ains	ein	an	one
Dwau	du	duo	duo	dau	tvai	zwei	twa	two
Tri	seh	treis	tres	tri	threis	drei	threo	three
Chatui	chehaur	tessares	quatuor	pedwar	fidvor	vier	feower	four
Pancha	pendj	pente	quinque	pump	finf	fünf	fif	five
Shash	shesh	hex	sex	chwech	saihs	sechs	six	six
Saptan	heft	hepta	septem	saith	sibun	sieben	scofen	seven
Ashta	hesht	okto	octo	wyth	ahtan	acht	eahta	eight
Navan	nuh	ennea	novem	nan	niun	neun	negon	nine
Dashan	deh	deka	deeem	deg	taihun	zehen	tyn	ten

That these Sanscrit terms should have been so widely diffused, while yet no traces of the Hindoo arithmetical notation should ever have been found outside of India, would seem to show that this derivation was antecedent to the formation of a written language, or, at least, prior to the invention of the *cipher.* A nomenclature may be easily transmitted orally by tradition ; a notation could be communicated and preserved only by records.

To the Sanscrit we are indebted for the denominations of our lowest two coins. From the Sanscrit *Sata* or *Shatum* (a hundred), through the Latin *centum*, we obtain our "cent ;" and from the Sanscrit *Dasa or Dashan* (ten), through the Latin *deeem* and the French *disme* or *dime*, we obtain the name of our "ten-cent piece."

The word *four* will give us "fo;" but for five, in order to avoid a conso-
nant recurrence, we shall have to resort to the original Sanscrit, *pancha*,
which will give us "pa." Our *six* will give us "si;" or "se;" but for
our next number, as we can derive no satisfactory help from English or
Latin, Greek or Sanscrit, we are driven to some arbitrary syllable. As
seven is the last of our series, we may accept the single independent term
with less reluctance ; and that its sound may be as distinctively marked
as possible, let us call it "ki."

Here, then, we have assigned for each of the numerals "a local habita-
tion and a name."

$$\mathsf{L}\ Un\ ;\ \mathsf{C}\ Du\ ;\ \mathsf{G}\ The\ ;\ \mathsf{F}\ Fo\ ;\ \mathsf{P}\ Pa\ ;\ \mathsf{G}\ Se\ ;\ \mathsf{H}\ Ki.$$

Our decades—twenty, thirty, forty— offer us the very suitable and sim-
ple suffix "ty" to designate our octades. Our hundred suggests the syl-
lable "der" as a convenient designation of the third place of figures ; and
our thousand will give us "sen." And here we may improve upon our
present mode of expressing "places" by employing these distinctive suf-
fixes as independent nouns, significant of a particular order of units, with-
out reference to any special or intrinsic value. Thus a simple unit would
indicate any figure occupying the first place ; a Ty would indicate any
figure occupying the second place ; a Der any figure occupying the third
place, etc.

But mindful of that prudent law—"economy of means"—and not to
burden our infant scheme with too great a load of unfamiliar nomencla-
ture (always the greatest obstacle to the reception of any novel system),
let us resort to combinations of these simple suffixes, instead of applying
a new term to each new place of figures. By this means we shall be re-
quired to introduce new terms only at the successive and advancing pow-
ers of each great unit. Thus using "Ty" for the second place, and
"Der" for the third place, we may very well employ the word "Ty-der"
for the fourth place, "Sen" for the fifth place, "Ty-sen" for the sixth
place, "Der-sen" for the seventh place, and "Ty-der-sen" for the eighth
place. Here is a pause ; and to do honor to the number *eight*, this should
comprise one independent period of figures ; to be followed by a new
term, the analogue of our Million.* We cannot derive a convenient suf-
fix, however, from this term ; we shall therefore have to coin a new one.
Let us call our great figure KALY. We have thus the progression : One
"Ty" squared is one "Der ;" one "Der" squared is one "Sen ;" one
"Sen" squared is one "Kaly," the intermediate places being expressed
by the obvious compounds of these words. Or to illustrate the series pro-
posed by our own decimal terms, it is as though having assigned eight places

*Our *Million*, the square of the Roman *Mille*, is a comparatively modern word ; and
useful as it is now universally esteemed, it appears on its first introduction to have met
with but little favor. "Bishop Tonstall, who has discussed at great length the Latin
nomenclature of numbers, speaks of the term *million* as in common use, but he rejects it
as barbarous" (*Peacock's Arithmetic*).

of figures instead of six for our million origin, we should reach it by this scale : Tens, hundreds, ten-hundreds, thousands, ten-thousands, hundred-thousands, ten-hundred-thousands, millions ; the "ten-hundreds" and the "ten-hundred-thousands" being interpolated places.

Words manufactured to meet a new want have always a somewhat barbarous and uncouth sound, until familiarized by custom ; and are usually received but slowly and with reluctance. Unless they can boast a pedigree and a history, they must expect from the world, like other parvenus, no very cordial greeting. From the habits of thought of a very large majority of mankind, it is found so much easier to use old words in a double sense, than to accept the precision of a new phraseology, that there is little doubt the octonary notation could be much more readily taught (except to children) by simply erasing the figures 8 and 9, from the common arithmetic. That it is more philosophical, however, to assign to everything its own appropriate name, can scarcely need a formal statement ; and if the system now proposed have the high claims and merits we have represented, no apology is required for the attempt to clothe it in a fitting garb. We here present accordingly the numeration table, as resulting from the names we have just above suggested :

NUMERATION TABLE.

ᒪ, Un = 1	ᒪᒪ, Unty-un = 9	Ϲᒪ, Duty-un = 17	Ɛᒪ, Thety-un = 25
Ϲ, Du = 2	ᒪϹ, Unty-du = 10	ϹϹ, Duty-du = 18	ƐϹ, Thety-du = 26
Ɛ, The = 3	ᒪƐ, Unty-the = 11	ϹƐ, Duty-the = 19	ƐƐ, Thety-the = 27
Ƒ, Fo = 4	ᒪƑ, Unty-fo = 12	ϹƑ, Duty-fo = 20	ƐƑ, Thety-fo = 28
P, Pa = 5	ᒪP, Unty-pa = 13	ϹP, Duty-pa = 21	ƐP, Thety-pa = 29
Ꮾ, Se = 6	ᒪꮾ, Unty-se = 14	Ϲꮾ, Duty-se = 22	Ɛꮾ, Thety-se = 30
Ᏼ, Ki = 7	ᒪᏼ, Unty-ki = 15	Ϲᏼ, Duty-ki = 23	Ɛᏼ, Thety-ki = 31
ᒪO, Unty = 8	ϹO, Duty = 16	ƐO, Thety = 24	ƑO, Foty = 32

Ƒᒪ, Foty-un = 33	Pᒪ, Paty-un = 41	Ꮾᒪ, Sety-un = 49	Ᏼᒪ, Kity-un = 57
ƑϹ, Foty-du = 34	PϹ, Paty-du = 42	ϹϹ, Sety-du = 50	ᏼϹ, Kity-du = 58
ƑƐ, Foty-the = 35	PƐ, Paty-the = 43	ꮾƐ, Sety-the = 51	ᏼƐ, Kity-the = 59
ƑƑ, Foty-fo = 36	PƑ, Paty-fo = 44	ꮾƑ, Sety-fo = 52	ᏼƑ, Kity-fo = 60
ƑP, Foty-pa = 37	PP, Paty-pa = 45	ꮾP, Sety-pa = 53	ᏼP, Kity-pa = 61
Ƒꮾ, Foty-se = 38	Pꮾ, Paty-se = 46	ꮾꮾ, Sety-se = 54	Ᏼꮾ, Kity-se = 62
Ƒᏼ, Foty-ki = 39	Pᏼ, Paty-ki = 47	ꮾᏼ, Sety-ki = 55	ᏼᏼ, Kity-ki = 63
PO, Paty = 40	ꮾO, Sety = 48	ᏼO, Kity = 56	ᒪOO, Under = 64

It will be seen by this table that we have no peculiar word corresponding to the "ten" of the denary scale ; and this is regarded as an

advantage, not only in being more systematic, but in giving greater precision of expression and idea. Instead of using the same word to indicate both a place, or local value (as the "ten-place") and a specific number, we are furnished with two distinct words—"Ty" designating the place and "Unty" specifying one in the ty-place, as "Duty" specifies two in the ty-place. All that is needed to carry out this system is to add a table of places.

NOTATION TABLE.

Units.	Ties.	Ders.	Tyders.	Sens.
ι Un	ι0 Unty	ι00 Under	ι000 Untyder	ι,0000 Unsen
ϲ Du	ϲ0 Duty	ϲ00 Duder	ϲ000 Dutyder	ϲ,0000 Dusen
Ɛ The	Ɛ0 Thety	Ɛ00 Theder	Ɛ000 Thetyder	Ɛ,0000 Thesen
Ϝ Fo	Ϝ0 Foty	Ϝ00 Foder	Ϝ000 Fotyder	Ϝ,0000 Fosen
Ρ Pa	Ρ0 Paty	Ρ00 Pader	Ρ000 Patyder	Ρ,0000 Pasen
Ƅ Se	Ƅ0 Sety	Ƅ00 Seder	Ƅ000 Setyder	Ƅ,0000 Sesen
Ꭸ Ki	Ꭸ0 Kity	Ꭸ00 Kider	Ꭸ000 Kityder	Ꭸ,0000 Kisen

Tysens.	Dersens.	Tydersens.	Kalies.
ι0,0000 Untysen	ι00,0000 Undersen	ι000,0000 Untydersen	ι,0000,0000 Unkaly
ϲ0,0000 Dutysen	ϲ00,0000 Dudersen	ϲ000,0000 Dutydersen	ϲ,0000,0000 Dukaly
Ɛ0,0000 Thetysen	Ɛ00,0000 Thedersen	Ɛ000,0000 Thetydersen	Ɛ,0000,0000 Thekaly
Ϝ0,0000 Fotysen	Ϝ00,0000 Fodersen	Ϝ000,0000 Fotydersen	Ϝ,0000,0000 Fokaly
Ρ0,0000 Patysen	Ρ00,0000 Padersen	Ρ000,0000 Patydersen	Ρ,0000,0000 Pakaly
Ƅ0,0000 Setysen	Ƅ00,0000 Sedersen	Ƅ000,0000 Setydersen	Ƅ,0000,0000 Sekaly
Ꭸ0,0000 Kitysen	Ꭸ00,0000 Kidersen	Ꭸ000,0000 Kitydersen	Ꭸ,0000,0000 Kikaly

The Unkaly is the eighth (or untieth) power of Unty. Its value is 16,777216; and it requires but one more figure to express this large amount, than is required by the denary scale. A second place of figures is not lost by our new system—that is, its notation does not exceed that of the common system by two places, until the number 8589,934592 is reached; these 10 figures requiring 12 (namely ι and eleven ciphers) in the octonary scale to represent their value. If this should appear surprising to any, it must be remembered, that although at 8, and at 64, an additional figure is required by the octonary system, yet after 10, and 100, the denary also requires this additional figure; and considering this, we shall find that the two scales are *equal* in the number of places occupied— from 1 to 7, inclusive—from 10 to 63, inclusive—from 100 to 511—from 1000 to 4095—from 10,000 to 32,767—from 100,000 to 262,143—from 1,000000 to 2,097151—from 10,000000 to 16,777215—from 100,000000 to

134,217727—and for the last overtaking, from 1000,000000 to 1073,741823. After this long-continued chase, the octonary scale at the next figure, or 1073,741824 (Under-Kaly) loses a place which is never regained. It may not be uninteresting to add, that this scale does not obtain an excess of *three* places until it reaches the enormous number of 9 trillions, 223372 billions, 036854 millions, 775808, these 19 figures being expressed by Ⴑ and 21 ciphers. This amount diminished by a single unit, or by the last figure 8 being exchanged for a 7, is expressed in the octonary system by 21 *kis* (Ⴜ) which would be an excess of only *two* places of figures.

Turning from this comparison of the relative powers of the two scales, to their relative simplicity, as exemplified by the octonary multiplication table, we shall find the contrast here as striking as was their parity on the other hand remarkable.

MULTIPLICATION TABLE.

	C	6	F	P	6	8
C	F					
6	6	ႪႩ				
F	ႪႩ	ႪF	C0			
P	ႪC	ႪႪ	CF	6Ⴑ		
6	ႪF	CC	60	66	FF	
8	Ⴑ6	CP	6F	F6	PC	6Ⴑ

The mere inspection of this table is sufficient to show, that the time and labor of acquiring it would not be half that required for committing to memory our received form; and this facility of acquisition would include almost a corresponding degree of readiness in its use. Figures, like furniture stored in the chambers of the brain, require a constant attention and arranging, to be kept in state for use; and the amount of care and trouble unconsciously bestowed upon them, must be proportioned to the number of the pieces after which we have to look. It is no idle boast, therefore, to say that a child could be taught a thorough knowledge of the four great rules of arithmetic, and a ready skill in their practical applica- tions, through the octonary system, in one-half the time required for obtaining an equal knowledge and skill by the common system. Nor is this simplification of arithmetical operations its only merit. The danger of error increases rapidly with the increasing complexity of the numeric

scale ; and there is no doubt that our new system would ensure an increase of accuracy, at least equal to its ratio of simplicity. And if to this were added the facility which would result from constructing all our tables of weight and measure upon this scale (a scale so admirably suited to them)—and thereby entirely discarding the whole tedious and troublesome practice of "reduction," from our Arithmetic—the economy of time and labor would be something quite astounding.*

Our exposition of the subject of numeration has been so extended that neither time nor space will now permit us to illustrate the practical working of the arithmetical system here proposed. It is evident, however, that we are here equipped with a mechanism fully adequate to the resolution and expression of all arithmetical operations. Framed by a strict analogy with our present system, it affords us every facility and advantage that this can boast ; and differing from it only in the number of its integers, it relieves us entirely from the difficulties and embarrassments which have ever been the opprobrium of our decimal scale. Merely to exhibit the form and method of our scheme, we may here indicate that the present year, "1887," would, in the octonary style be expressed ЄРЄᗺ—*Thety pader and thety-ki*. The diameter of the earth (7925 miles) would be expressed ᒪᗺЄᕼР—*Unsen, Kity theder sety-pa ;* or in feet (41,847,188) ᑕ,ЄᕼРᑎ,ᖴᕼᑕᖴ— *Dukaly, thety kider patysen, foty seder duty-fo.*

We now proceed as rapidly as possible to the application of this improved numeration to the determination and distribution of a system of weights and measures. Of all the systems of metrology yet perfected, or even proposed, that of the French is, in the philosophical character of its standards, as well as in the ingenuity, simplicity and precision of its details, undoubtedly by far the most admirable and the most worthy of our imitation. "The French System," says Mr. Adams in the excellent Report on Weights and Measures from which we have already more than once had occasion to quote, " embraces all the great and important principles of uniformity which can be applied to weights and measures. *But that system is not yet complete ; it is susceptible of many modifications and improvements.* Considered merely as a labor-saving machine, it is a new power offered to man incomparably greater than that which he has acquired by the new agency which he has given to steam. It is in design the greatest invention of human ingenuity since that of printing. But

* "It is impossible to estimate with any degree of accuracy," says Mr. Nichol, "the amount of labor annually thrown away by the nation at large, while persisting in performing the manifold computations necessary to its gigantic commerce and industry, by means of a series of tables so needlessly complicated and imperfect as those now in use. But the waste of time and loss of money must be something quite enormous, while every day it becomes greater and greater. Were the different denominations of weights, measures and money brought into harmony with the fundamental principle of our common arithmetic, it may be safely affirmed that the labor.of commercial and professional calculations would be reduced much below one-half of what is now expended in this direction, while the risk of errors would be diminished in a still greater ratio" (*Encyclopedia of the Physical Sciences*, art. "Weights and Measures," page 778).

like that and every other useful and complicated invention, it could not
be struck out perfect at a heat. Time and experience have already dic-
tated many improvements of its mechanism. But all the radical principles
of uniformity are in the machine.   *   *   *   *Uniformity* of weights and
measures—permanent, universal uniformity, adapted to the nature of
things, to the physical organization and to the moral improvement of man
—would be a blessing of such transcendent magnitude, that if there
existed upon earth a combination of power and will adequate to accom-
plish the result by the energy of a single act, the being who should exer-
cise it would be among the greatest of benefactors of the human race.
The glory of the first attempt belongs to France.   France first surveyed
the subject of weights and measures in all its extent and all its compass.
France first beheld it as involving the interests, the comforts and the mor-
als of all nations, and of all after ages.   *   *   * In freely avowing the
hope that the exalted purpose first conceived by France may be improved,
perfected and ultimately adopted by the United States and all other
nations, equal freedom has been indulged in pointing out the errors and
imperfections of that system, which have attended its origin, progress
and present condition.''

Looking at the French *metre* simply as a practical material standard, the
first criticism we would naturally have to make upon it, is that it gives us
a measure most unfortunate in its size.

In selecting a standard of measure (without any reference to its ideal
derivation) two considerations of very obvious and primitive notice im-
pose a tolerably definite limit as to what should constitute the length of a
useful, popular measuring rule. The first is that it should be conveniently
portable,* if not in a pocket, at least in a satchel, or upon the thigh ; the
second is that when held by one hand in careful and precise position for
taking or giving measures its two ends should each be distinctly within
accurate view, and within easy reach of the free hand for minute mark-
ing without any constraint or effort of the body. These two conditions,

*"Perhaps for half the occasions which arise in the life of every individual for the
use of a linear measure, the instrument to suit his purposes must be portable and fit to
be carried in his pocket * * * For all the ordinary purposes of mensuration, except-
ing itinerary measure, the metre is too long a standard unit of nature. It was a unit
most especially inconvenient as a substitute for the foot, a measure to which, with
trifling variations of length, all the European nations and their descendants were accus-
tomed. The foot-rule has a property very important to all the mechanical professions
which have constant occasion for its use ; it is light and easily portable about the per-
son. The metre, very suitable for a staff, or for measuring any portion of the earth, has
not the property of being portable about the person ; and for all the professions con-
cerned in ship or house building, and for all who have occasion to use mathematical in-
struments, it is quite unsuitable. It serves perfectly well as a substitute for the yard or
ell, the fathom or perch, but not for the *foot*. This inconvenience, great in itself, is made
irreparable when combined with the exclusive principle of decimal divisions. The
union of the metre and of decimal arithmetic rejected all compromise with the foot.
There was no legitimate extension of matter intermediate between the ell and the palm,
between forty inches and four. This decimal despotism was found too arbitrary for en-
durance '' (*Adams's Report on Weights and Measures*).

which would both be assigned on perhaps one-half the occasions of its familiar use, render it tolerably manifest that its length should be not less than twelve inches, and while certainly excluding the yardstick and the metre, would probably designate the carpenters' two-foot rule as reaching the maximum limit of practicable length. Both the French metre and our yardstick are very awkward and inconvenient standards, being too long for all ordinary purposes of mensuration, excepting itinerary mea- sure, and as a popular standard utterly worthless except on the counter of the draper. Moreover, we would naturally select such a rule as we would measure our houses by, or the furniture within them ; such a rule as the carpenter would cut off or lay off his boards by ; such a rule as the mechanic could use in his workshop or the machinist handle in fitting his engines. Theoretically it matters little whether our unit of reference be the inch or the mile, but for the practical business of daily life it becomes a matter of the very highest importance that our unit of measure should be such a one as shall have the most convenient and universal appli- cation.

Two standards only have ever had a general use and currency—the *cubit* and the *foot*. Both derived from the human person, it is natural they should be found the most useful measures for the common wants of the person. The cubit may be said to be almost a natural standard ; and it is the most ancient of measures, while it is still prevalent through- out the orient. Universal, or nearly so, throughout the nations of an- tiquity—it was the common measure of the Israelites, and is referred to in their earliest records. The ark is measured in cubits (Gen. vi, 15), and the height of the flood is in cubits. Goliath's height was six cubits and a span. The temple of Solomon is measured in cubits ; and walls of cities are measured by the same (2 Kings xiv, 13). The foot appears to be a much later standard of measure. Introduced by the Greeks and Romans, it has prevailed in modern times wherever the Roman influence has been felt.

If the foot has been found a more manageable multiple of both the pace and the fathom or its half—the ell—than the cubit, we are disposed to regard the latter as the more beautiful and useful rule, and the more con- venient unit of length. Certainly the occasions are not unfrequent, when we need the addition of a few inches to our foot-rule to measure common objects. At all events, in selecting a standard, adapted to the popular wants, it may be regarded as tolerably manifest that its length should not be less than a foot, and that it should not exceed two feet—the common carpenters' rule. The cubit is the mean between these extreme limits.

This consideration brings us to the derivation of the standard. "In all the proceedings," says Mr. Adams, "whether of learned and philosophical institutions, or of legislative bodies, relating to weights and measures within the last century, an immutable and invariable standard from nature, of linear measure, has been considered as the great desideratum for the basis of any system of metrology. It is one of the greatest merits

of the French system to have furnished such a standard for the benefit of all mankind. * * * In the establishment of the French system, the pendulum, as well as the meridian, has been measured ; but the *standard* was, after a long deliberation, after a cool and impartial estimate of the comparative advantages and inconveniences of both, definitively assigned to the arc of the meridian, in departure from an original prepossession in favor of the pendulum." A writer in the *Edinburgh Review* for January, 1807, remarks: "Three different units fell under the consideration of these philosophers, to wit, the length of the pendulum, the quadrant of the meridian, and the quadrant of the equator. If the first of these was to be adopted, the commissioners were of opinion that the pendulum vibrating seconds in the parallel of 45 degrees deserved the preference, because it is the arithmetical mean between the like pendulums in all other latitudes. They observed, however, that the pendulum involves one element which is heterogeneous, to wit, time ; and another which is arbitrary, to wit, the division of the day into 86,400 seconds. It seemed to them better that the unit of length should not depend on a quantity, of a kind different from itself, nor on anything that was arbitrarily assumed. The commissioners therefore were brought to deliberate between the quadrant of the equator, and the quadrant of the meridian ; and they were determined to fix on the latter, because it is most accessible, and because it can be ascertained with the most precision" (*Edinburgh Review*, Vol. ix, p. 379).*

That this selection was wise at the time it was made, cannot be doubted. That it would be wiser now to select the equator, can, perhaps, be made equally evident. By the modern methods of electro-magnetic determination of longitude, an arc of the equator could now be ascertained with as much accuracy, as one of a meridian, and perhaps with even greater precision. A national, or what would be far nobler, an international commission, liberally endowed with every needed equipment, for measuring in South America, and in Africa, arcs of the equator—if possible entirely across either continent; and also (what would be very important) one through the opposite island Borneo—is an enterprise due to the enlightened spirit and scientific progress of the age, and would be one worthy of the united wisdom and resources of the three greatest nations of the world. The determination of the precise figure and dimensions of our globe—that fundamental problem of practical astronomy—is one of such transcendent importance, that no outlays should be regarded as injudicious or misapplied that would offer the prospect of even a slight improvement in the accuracy of our results.

The equator is, in the first place, undoubtedly the true girth and measure of the earth ; and the circumference should always be understood to be this natural measure, unless otherwise specified. In the next place, the meridian not being a circle (owing to the polar flattening of the earth) no two degrees of its quadrant have exactly the same value ; which renders the estimates of its degrees exceedingly awkward. According to the com-

* See note C, page 360.

putations of Mr. T. J. Cram (*Silliman's Journal of Science* for 1837, Vol. xxxi, page 230), one degree of latitude at the pole is equal to 69.39759375 miles, while one degree at the equator is only 68.70859375 miles—a differ ence of more than two-thirds of a mile ! In addition to all this there is some reason for doubting whether different meridians are uniform in length and curvature. An arc of the meridian south of the equator, measured in 1752, by Lacaille (at the Cape of Good Hope) gave very un satisfactory results.

But through the reductions of various eminent mathematicians we have now the equatorial circumference of the earth as well and accurately determined as any other measure of it. The two best and most recent determinations of the earth's equatorial diameter, are those of Bessel and Airy, who, by independent calculations, agree in the value 7925.6 miles, and differ only by 234 feet ! Bessel making it 41,847,192 feet, and Airy, 41,847,426 feet. The mean of these results will give us 131,467,196 feet, as probably a very close measure of the earth's equator. We have every reason, therefore, for deducing our standard of measure from this line— the only true circle by which the earth is circumscribed ; we have none for going back to the irregular meridian.

In no particular has the decimal principle of the French system proved so signal and utter a failure as in its application to the division of the cir cle. We have already noticed that the sixth part of the circle is one of its fundamental divisions—one which cannot be neglected for any theoretical advantage of adherence to system. We have seen, moreover, how admi rably our present division of the quadrant into ninety parts or degrees answers all the various purposes required. In adding ten more degrees to the whole, so as to make an even hundred, the French philosophers sacri ficed completely its primary and beautiful relations. The sextant no lon ger had a possible expression in the centesimal scale. A very brief exper iment demonstrated what should have been clearly anticipated without it, that the new degrees were wholly impracticable. This part of the system was therefore speedily and universally abandoned,* and yet this was really a surrender of the very foundation of the metrical division.

The *metre* had been made the 10 millionth part of the quadrant, that the new degree might represent just 10 myriametres ; but the abolition of this ideal degree left the myriametre (and with it of course the metre) a most inconsequential and unmeaning unit. So that now the kilometre no lon ger represents a minute, and the decametre a second, as was its original plan and purpose.

The selection of the meridian necessarily involved a reference to its nat ural fraction, the quadrant—the distance from pole to equator ; but had

* " The new metrology of France, after trying it [the principle of decimal division] in its most universal theoretical application, has been compelled to renounce it for all the measures of astronomy, geography, navigation, time, the circle and the sphere ; to modify it even for superficial and cubical linear measure, and to compound with vulgar fractions in the most ordinary and daily uses of all its weights and all its measures " (*Adams's Report*).

the equator been the standard chosen, inasmuch as it has no such natural
measure, the sextant of it might just as properly have been made the
starting unit as its quadrant. And this would have escaped the principal dif-
ficulty ; for the sextant will easily supply us with a multiple of the quadrant,
though the latter may not conversely so readily commensurate the former.·
Instructed by such distinguished failures, let us then start with the sex-
tant of the equator as our prime unit of measure. We shall thus be able
to select a final modulus or rule, mainly with reference to its most desira-
ble length—no longer trammeled by the compounding of binary and ter-
nary divisions. Ten million metres made the quadrant. Our octonary
scale is also furnished with its grand unit (the eighth power of the octade),
which for want of a better name we have christened *unkaly* (Ⳑ,0000,-
0000). The sextant of the equator is 21,911199½ feet, or 262,934392
inches. This divided octavally into *unkaly* parts, gives us the quotient,
15⅔ inches, *almost exactly our ideal measure!* Midway between the two
great rival standards of olden time, the cubit and the foot, it seems the
very compromise of differences, the harmonizer of conflicting systems,
and supplies us with a "module" perfectly suited to every requirement of
popular mensuration. It needs but the application of octonary multiples
to complete a metrology simple and unexceptionable.

Before giving the table, however, it will be proper to suggest a slight
modification in the divisions of the circle, as a subject controlling, to some
extent, the details of our linear measures. Should the degree retain its
present value as the 360th part of the circle, we should advocate strongly
the employment of this unit of the equatorial circle, as the origin of our
new standard of measure. Dividing the degree into *undersen* parts
(Ⳑ00,0000), we should have a module about one inch longer than that
previously obtained, and somewhat nearer, therefore, to the ancient cubit.
Its exact length would be 16.717 inches.

The number 60, however, approaches so near to the octonary *under*
(64) that the temptation would be very strong to reduce degrees, minutes
and seconds to the simplicity of the general notation, unless there ap-
peared some strong reason for retaining the present sexagenary scale. But
there is no special occasion for dividing small arcs into thirds or sixths,
that gives this ancient and venerable system any advantage comparable
to that we should have of adding up or subtracting degrees, minutes and
seconds by a single operation, instead of resorting as now to reduction.
On the contrary, the need of frequent binal division is here, as with other
values, very apparent ; and in this respect the number 60 is very defec-
tive, as it permits but two bisections. The mariners' compass affords us a
good illustration of the convenience experienced in a continued bisection
of angles.* There would therefore be a positive benefit in substituting

* The cardinal points dividing the circle into quarters—each quadrant is divided into
halves or octants, each octant into halves and quarters called "rhumbs" or "points"
(8 in the quadrant),and finally each of these points into halves and quarters ; the rhumb
or point being 11° 15', and the quarter rhumb or point 2° 48' 45".

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the number 64 for 60. This would interpolate 4 degrees into the sextant, or 6 degrees into the quadrant ; making the right angle to be expressed by 96° instead of 90° as at present. This, then, is the table we should propose ; in which, it will be seen, the present values of arc are not so altered as to disturb appreciably our long-established ideas of degree, minute and second.

DIVISIONS OF THE CIRCLE.

L00	(64)	tertials	make	1 second	=	0″.823974
L00	(64)	seconds	"	1 minute	=	52″.734375
L00	(64)	minutes	"	1 degree	=	$(\frac{15}{16})°$ or 56′15″
L00	(64)	degrees	"	1 sextant	=	60°
LF0	(96)	degrees	"	1 quadrant	=	90°
E00	(192)	degrees	"	the semicircle =		180°
600	(384)	degrees	"	the circle	=	360°

One obvious advantage of this scale, in addition to its simplification, would be to bring the azimuth compass into harmony with the mariners' compass, by giving them common measures. As the latter divides the quadrant into 8 "points" or "rhumbs," each of these would be LF° (12 degrees) instead of 11° 15′, as at present ; and the quarter-point would be E° (3 degrees) instead of 2° 48′ 45″.

The zodiac, or ecliptic circle, has from time immemorial been divided into 12 "signs." This would be found a very convenient unit to be applied to such arcs generally, as would also the smaller unit of its quarter, or L0°, the eighth part of the sextant. As there is no name for this, let us give it the name of "arc" (made technical and specific), a name not inappropriate, since it is about the smallest arc we can readily distinguish from a straight line. This would give us the following scale :

L0°	(8 degrees)	=	1 arc	=	7° 30′
F0°	or 4 arcs	=	1 sign	=	30°
60°	" 6 arcs	=	1 octant	=	45°
L00°	" 8 arcs or 2 signs	=	1 sextant	=	60°
LF0°	" 12 arcs or 3 signs	=	1 quadrant	=	90°

Should the above scheme of graduation for the circle be accepted, it

will give an admirable simplicity to our table of lengths, which without further preface is herewith subjoined :

TABLE OF LINEAR MEASURE.

		=	value				decimal
		1 point	=	($\frac{1}{200}$ inch nearly)				0.0038 ins.
LO (8) points	make 1 line	=	($\frac{1}{33}$ " ")				0.03 "
LO (8) lines	" 1 dent	=	($\frac{1}{4}$ " ")				0.245 "
LO (8) dents	" 1 digit	=	(2 inches ")				1.959 "
LO (8) digits	" 1 MODULE	=	($15\frac{2}{3}$ " ")			1 ft.	3.672 "
LO (8) MODULES	" 1 rod	=			3 yds.	1 ft.	5.37 "
LO (8) rods	" 1 chain	=			27 yds.	2 ft.	7. "
LO (8) chains	" 1 furlong	=			222 yds.	2 ft.	8. "
LO (8) furlongs	" 1 mile	=		1 mile,	23 yds.	0 ft.	5. "
LO (8) miles	" 1 league	=		8 miles,	1 yds.	0 ft.	3. "
LO (8) leagues	" 1 *degree*	=		64 miles,	1480 yds.	2 ft.	5. "
LO (8) *degrees*	" 1 *arc*	=		518 miles,	1286 yds.	1 ft.	11. "
LO (8) *arcs*	" 1 *sextant*	=		4,149 miles,	1493 yds.	0 ft.	4. "
6 (6) *sextants*	" the *circumference* =		24,899 miles,	158 yds.	2 ft.	

The table of lengths proper terminates with the league ; the denominations following being those of arc. From the derivation of the standard, however, they coincide with precise measures, and are therefore properly included in the table.

The "point" gives a dimension about equal to that of a section of a human hair, or of a very fine grain of sand, and may be considered about the limit of visible magnitude. It is therefore a very suitable origin of linear value, while it is an equally appropriate point of departure for microscopic measurements. The "dent" and the "digit" would be convenient measures for small articles. While this new *metre* gives us one of the most convenient rules that can be devised, the "rod" furnishes us with a highly useful ten-and-a-half foot measuring pole, and eight times this measure gives us the best "chain." But the peculiar beauty of the new Module is, that it precisely corresponds with the *tertial* of the new degree. *Under* Modules make one second (the "chain") ; *Under* seconds make one minute (the "mile ") ; *Under* minutes make one degree ; and *Under* degrees—the Sextant. Or, progressing by the successive squares—*Unty* Modules make the rod ; *Under* Modules make the chain ; *Unsen* Modules make the mile ; *Unkaly* Modules make the Sextant.

As referred to the French measures, we have for the value of our principal new denominations the following : the "line " = 0.77746 *millimetres;* the "dent " = 6.21975 *millimetres ;* the "digit " = 4.9758 *centimetres ;* the

"module" $= 3.98064$ *decimetres ;* the "rod" $= 3.18451$ *metres ;* the
"chain" $= 2.54761$ *decametres ;* the "furlong" $= 2.03809$ *hectometres ;*
the "mile" $= 1.63047$ *kilometres ;* and the "league" $= 1.30437$ *myria-
metres.*

For those measures in most common use, that is for those clustering
immediately around the Module, it would doubtless be found highly con-
venient to give denominations to the halves and quarters ; and thus con-
form them to the universal popular tendency to binary divisions.   We
therefore propose the following supplementary table ; not to be on any
account incorporated with the preceding, nor in any respect to modify it ;
but to retain always its subordinate character.

2 dents	make 1 nail		0.48975 ins.	
2 nails	" 1 inch		0.9795	"
2 inches	" 1 digit		1.959	"
2 digits	" 1 hand		3.918	"
2 hands	" 1 span		7.836	"
2 spans	" 1 Module		15.672	"
2 Modules	" 1 ell	2 ft.	7.344	"
2 ells	" 1 fathom	3 ft.	2.688	"
2 fathoms	" 1 rod	10 ft.	5.376	"

Our tables of area, or surface measure, would of course be derived
directly from our linear measures, by the familiar law of squares.

TABLE OF SQUARE MEASURE.

LO (8) digits square, or LOO (64) square digits, make 1 square Module:
LO (8) Modules " " LOO (64) " Modules, " 1 " rod:
LO (8) rods " " LOO (64) " rods, " 1 " chain:
LO (8) chains " " LOO (64) " chains, " 1 " furlong:
LO (8) furlongs " " LOO (64) " furlongs, " 1 " mile.

For popular purposes, however, it would be necessary, or convenient, to
have more numerous denominations of area measure ; and a less rapid
progression than that of *unders,* given in the above merely geometrical

table of perfect squares. We therefore propose to insert intermediate values, so as to give our table the systematic or octonary form.

<p style="text-align:center">TABLE OF AREA—OR SURFACE MEASURE.</p>

			Mile	Acres	Yards	Feet	Inches.
1 sq. Module	=					1	101.615
L0 (8) square Modules, make 1 *lot,*	=				1	4	92.92
L0 (8) *lots,* " 1 sq. rod,	=				12	1	23.364
L0 (8) square rods, " 1 *plat,*	=				97	0	42.9
L0 (8) *plats,* " 1 sq. chain,	=				776	2	55.3
L0 (8) square chains, " 1 *acre,*	=			1	1370	1	10.42
L0 (8) *acres,* " 1 sq. furlong,	=			10	1280	8	83.4
L0 (8) square furlongs, " 1 *district,*	=			82	567	5	91.27
L0 (8) *districts,* " 1 sq. mile,	=		1	16	4541	0	10.

The intermediate (alternate) denominations of this table are not *perfect squares;* hence it was thought more correct to assign terms to them indicative of their superficial character without the use of the prefix "square." We observe here one advantage that would result from the radix of numeration being a perfect square. The square root of 8 is 2.828427124; or $\sqrt{\text{L0},} = \text{C.6POLLALFG}$; hence this value would represent in any given units, the side of a square equal to L0 (8) of the square units. Thus the side of a square "lot" would be C *Modules,* 6 *digits,* P *dents,* 0 *lines,* and L *point.* The side of a square "plat" would be C *rods,* 6 *Modules,* P *digits,* 0 *dents,* L *line,* and L *point.* The side of a square "acre" would be C *chains,* 6 *rods,* P *Modules,* 0 *digits,* L *dent,* L *line,* and H *points.* And the side of a square "district" would be C *furlongs,* 6 *chains,* P *rods,* 0 *Modules,* L *digit,* L *dent,* H *lines,* and L *point.* A very simple parallelogram is however afforded us, which gives with precision the dimensions of these respective areas. Thus a "district," as a land measure, is a rectangular space of ground, measuring two furlongs in one direction, and four furlongs in the other; an "acre" a similar space of ground measuring two chains in one direction by four chains in the other; a "plat," a space measuring two rods in one direction, by four rods in the other; and a "lot" is in like manner a surface of two Modules by four Modules. This table presents, therefore, the simplest ratios of superficial measure which could be devised; and would be found admirably adapted to every purpose of mensuration. For smaller surfaces, it is probable that the following supplementary table would prove a useful resort:

							Yards	Feet	Inches.
4 square dents			make 1 square nail			=		· ·	0.239858
4	"	nails	"	1	"	inch	=			0.959433
4	"	inches	"	1	"	digit	=			3.837735
4	"	digits	"	1	"	hand	=			15.350941
4	"	hands	"	1	"	span	=			61.403766
4	"	spans	"	1	"	Module =		1	101.615
4	"	Modules	"	1	"	ell	=		6	118.460
4	"	ells	"	1	"	fathom =	3	0	41.841
4	"	fathoms	"	1	"	rod	=	12	1	23.364

For measuring volume, we would naturally employ simply the cubes of the preceding denominations; while the contents of such cubic metres respectively, of distilled water at its maximum density, would as obviously furnish the measures of weight. Throughout these derivative tables, we propose to adopt the MODULE as the universal standard. In this respect our linear unit is very greatly superior to the *Metre*, which, from its inconvenient size, has been made practically a standard only of lengths. The *Are* (the unit of surface) is derived, not directly from the *Metre*, but from the *Decametre;* the *Litre* (the unit of capacity) is derived from the cube of the *Decimetre;* and lastly, the *Gramme* (the unit of weight) is derived from the cube of the *Centimetre.* The greater simplicity of our project is manifest in this contrast.

TABLE OF VOLUMES.

								Cubic Feet.	Cubic Inches.
					1 cubic dent		=		0.01468
10 (8) cubic dents			make 1	"	nail		=		0.11747
10 (8)	"	nails	"	1	"	inch	=		0.93977
10 (8)	"	inches	"	1	"	digit	=		7.51817
10 (8)	"	digits	"	1	"	hand	=		60.14537
10 (8)	"	hands	"	1	"	span	=		481.16296
10 (8)	"	spans	"	1	"	MODULE	=	2.22760	
10 (8)	"	MODULES	"	1	"	ell	=	17.82085	
10 (8)	"	ells	"	1	"	fathom	=	142.56680	
10 (8)	"	fathoms	"	1	"	rod	=	1140.53441	

This simple scale of volumes or bulks, derived directly from our smaller linear table, gives a good illustration of the great beauty and convenience flowing out of the employment of a radix of numeration which is a perfect cube. Each of the cubic measures of the above table has for the dimensions of its side two of the linear values above it.

The practical conveniences of simple and direct relations between lengths, weights, and measures of capacity are certainly too obvious and too great, to be lightly thrown away. Thus, where we are furnished with a measure, the root of whose cube is precisely a measuring rule in common use (one of the many advantages which result from an octonary scale of weights and measures), the benefit is by no means trivial ; the farmer can always, without any calculation, make himself a cubical box (whether to supply, or to verify a measure) whose capacity shall be fully as accurate as the "bushel" he may purchase—even admitting that such a process may not have the precision that would satisfy the experimental philosopher. And this is a benefit which would attach equally to every unit of measurement in the scale. Whenever so radical a change is contemplated as the introduction of new divisions or denominations of measure, the importance of adopting at the same time the most useful or convenient standards that can be devised, is too eminent to justify a moment's hesitation in throwing aside everything that has not some intrinsic value to plead for its preservation.

TABLE OF DERIVATIVE MEASURES.

The cubic	dent	gives the	morsel	measure	and the	carat	weight.
"	nail	" "	ligule	"	"	scrap	"
' "	inch	" "	cup	"	"	semy	"
" "	digit	" "	gill	"	"	unce	"
' "	hand	" "	quart	"	"	libra	"
' "	span	" "	octa	"	"	stone	"
" "	MODULE	" "	MODIUS	"	"	PONDUS	"
" "	ell	" "	pipe	"	"	ton	"
" "	fathom	" "	butt	"	"	load	"
" "	rod	" "	hold	"	"	keel	"

This table furnishes us with a complete system. It needs but a simple calculation to exhibit our weights and measures in full. Our measures of capacity with their respective values are as follows :

Table of Capacity Measure.

						Galls.	Pts.	Oz.	Drs.	Minims.
				1 parvum	=					.488
10	(8)	parvums	make	1 morsel	=					3.905
10	(8)	morsels	"	1 ligule	=					31.214
10	(8)	ligules	"	1 cup	=				4	9.955
10	(8)	cups	"	1 gill	=			4	1	19.64
10	(8)	gills	"	1 quart	=		2	1	2	37.
10	(8)	quarts	"	1 octa	=	2	0	10	4	56.
10	(8)	octas	"	1 Modius	=	16	5	4	7	35.
10	(8)	Modiuses	"	1 pipe	=	133	2	7	4	41.
10	(8)	pipes	"	1 butt	=	1066	3	12	5	56.
10	(8)	butts	"	1 hold	=	8531	6	5	7	28.

Our language is unfortunately but very poorly supplied with terms expressive of capacity ; and as the existing names for the smaller liquid measures used by the apothecary ("fluid-drachm" and "fluid-ounce") are exceedingly objectionable, from their reference to the incongruous standard of weight, we are compelled to reject them, although we have no appropriate denominations to substitute. The word "morsel" is perhaps sufficiently indeterminate to answer the purpose ; and the Roman *ligula*, a small measure of about a spoonful, supplies a convenient term, having the same recommendation. The "cup," which is equally indefinite, represents about a half-ounce. The *Modius* of the Romans was about the quarter of a bushel ; the term has been selected as a suitable one for indicating a standard *measure*, and also as suggesting its dimension, as the cube of the Module. The circumstance that it is here applied to a much larger volume than it was originally is comparatively unimportant.

As referred to our common table of "dry measure," as it is called, the new "quart" is equal to 1.79 pints ; the "octa" is equal to 7 quarts and one-third of a pint, or about one-twelfth less than a peck ; the new "Modius" contains 3849.3 cubic inches, and is therefore equal to one bushel, 3 pecks, 1 quart and half a pint, or to very nearly 1⅘ bushels, the U. S. bushel containing 2150.4 cubic inches ; the new "pipe" is equal to 14 bushels, 1 peck, 2 quarts and half a pint, and the new "butt" is equal to 114 bushels, 2 pecks and 2 quarts.

In the French measures our "quart" is very nearly equal to the *litre*, being .9855 of a *litre ;* our "octa" = 7.884 *litres*, and our "Modius" = 63 *litres*.

It may not be out of place to mention here (as exhibiting an interesting and very early anticipation of our octonary scale of measures in England)

that by the act of 51st Henry III (1266), it was declared that "8 pounds [of wheat] do make the gallon of wine, and 8 gallons of wine do make a London bushel, and 8 London bushels do make the quarter."

Our proposed system of weights forms but a corollary from the preceding table of capacity measures ; a Modius of pure water forming the standard unit, which we therefore call our *Weight* or *Pondus*. Taking the value of the cubic inch of distilled water at maximum density at 252,745 grains (the weight adopted by Mr. Hassler for the U. S. standard), the Modius or cubic Module would weigh 972891.328 grains, or 138 pounds, 15 ounces, 329.22 grains avoirdupois. This will give us the following table :

TABLE OF WEIGHTS.

					Av. lbs.	Oz.	Grains.
		1 mite	=				0.464
L0	(8) mites	make	1 carat	=			3.711
L0	(8) carats	"	1 scrap	=			29.69
L0	(8) scraps	"	1 semy	=			237.52
L0	(8) semies	"	1 unce	=		4	150.178
L0	(8) unces	"	1 libra	=	2	2	326.42
L0	(8) libras	"	1 stone	=	17	5	423.91
L0	(8) stones	"	1 PONDUS	=	138	15	329.22
L0	(8) PONDUSES	"	1 ton	=	1111	14	8.76
L0	(8) tons	"	1 load	=	8895	0	70.08
L0	(8) loads	"	1 keel	=	71160	1	123.14

While the "Pondus" is the standard of determination, the "libra," as the unit of weight in most common use, would be the secondary or derivative standard. Since *Under* "libras" make the "Pondus," this corresponds to our present hundred-weight. The new "ton" is not quite half a ton, and the "load" is very nearly 4 tons.

The "keel" is one-half larger than the English keel (a weight used only for coal), which is equal to 21 tons 4 cwt., and of which twenty make a "ship load." Or the English keel is two-thirds of our "keel," as above given.

Estimated by the French weights, our "scrap" = 1.924 *grammes ;* our

"semy" $= 1.5393$ *decagrammes;* our "unce" $= 1.2314$ *hectogrammes,* and our "libra" $= .98514$ kilogramme.

It would probably be found convenient to distribute the more popular or frequently used weights (those from the "scrap" to the "libra") upon the binary scale ; but as the divisions of halves and quarters practically accomplish this, it seems hardly necessary to suggest a series of intermediate denominations.

In the new standard of length here proposed and developed, we believe that every excellence of the French standard has been carefully preserved, and all its imperfections as successfully avoided. Starting from the same general principles by which that was obtained, we have made no departure from the details of its derivation, not required by the plainest and soundest deductions of experience, philosophy and common sense. Does the French method propose an *aggravated* yard as a convenient unit, we show the superiority of the cubit. Does it (on good grounds at the time) select an elliptical meridian, as its origin of measure, we show still better grounds for preferring the equatorial circle. Does it look (almost necessarily) to the quadrant as a natural unit, we show the greater propriety of the sextant. Does it rest on a thoroughly decimal basis, we show the most cogent reasons for adopting an octonary distribution. Does it find a fitting divisor only in the seventh power of its decimal radix, we accidentally find it in a great arithmetical unit—the eighth power of the octade. Does it finally give as its finished product, an imperfect *Metre,* we offer for acceptance a perfect *Module.*

The system of metrology derived from this new standard has in it nothing that is arbitrarily assumed. Each part of it is dependent upon every other, and each part flows from each, by a logical and systematic necessity. The whole is thus a perfect unit, simple and complete—comprehending every relation of dimension and of weight, and adequate to every purpose of precision, the minutest as well as the grandest.

We have thus endeavored to unfold with as much conciseness as was compatible with a clear presentation of the subject, what is regarded as the best possible method of fulfilling all the varied and difficult conditions required in an acceptable system of weights and measures, as well as the most effectual means of promoting that great desideratum of international commerce, an ultimate uniformity of standards among the nations of the earth. The serious and radical defects of our existing systems have been briefly noticed, and from the experience thus acquired the essential and practical wants of the community have been incidentally pointed out. As the result of this investigation, it is believed that there is no other practicable solution of the problem ; for the attainment of a real uniformity, there seems to be no other process or alternative. No disadvantage would follow the adoption of this plan, save that of the disturbance and confusion necessarily consequent upon every change, and which must form the price of every valuable reform.

If it be urged that the introduction of still another system of weights

and measures, and one having no common unit with either the French or the English system, would be only adding to the existing diversity of standards, instead of tending to that great scheme of uniformity so cherished by the philanthropist, we have to reply that, if the system proposed be really of all the best adapted to the needs not only of one, but of all nations, then is the prospect of a general uniformity most reasonably to be anticipated *from* its introduction. If neither the metrology of England (which is also ours), nor yet that of France, is ever likely to obtain a universal conquest, some better scheme alone remains to give us a hope of ultimate success. Such a scheme is here presented. Founded upon the simplest and yet most comprehensive basis, it contains nothing that could be regarded as in any respect peculiar to one locality or latitude, or more suitable for one nation than for any other. Encumbered by no abstruse nomenclature, it aims at no superfluous verbal uniformity, but leaves each people to employ such designations of its units as may appear to each most easy and familiar.

Mr. Adams, after his unequaled analysis of the English system of measures, in view of its close agreement with our own, discountenances all attempts at a premature innovation. Without approving in his report of the introduction of the French system, he thinks it would afford the best prospect of securing "uniformity;" and remarks, "were it even possible to construct another system on different principles, but embracing in equal degree all the great elements of uniformity, it would still be a system of diversity with regard to France, and all the followers of her system. And as she could not be expected to abandon that which she has established at so much expense, and with so much difficulty, for another possessing, if equal, no greater advantages, there would still be two rival systems with more desperate chances for the triumph of uniformity."

On the contrary, it is believed, that provided a new system could be framed, which *had* demonstrably "greater advantages" than her own, France would be among the first of nations to hail its advent and to welcome its adoption. A nation to which belongs the honor and the glory of having been the first to invite the fraternal co-operation of other powers, and the first to work out with unwearied science, skill and labor, a comprehensive organization of that ideal metrology—unrivaled in its philosophy and symmetry—cannot be the last to appreciate any real improvement of that economy ; or to submit to any sacrifice which should promote the realization of such improvement. Nor could the entire abandonment of that which has cost so much be accounted too great a sacrifice, if only through it could be accomplished that magnanimous design to which it owed its origin. It would have to be looked upon as a costly but invaluable experiment—as a great and necessary progression to an end, by which alone was rendered possible any higher attainment. The system here elaborated is but a development of *that*.

A project which contemplates the entire subversion of the existing

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arithmetic, with its immense stores of fact and formula, is certainly a most startling proposal ; and is one which will doubtless be regarded by the majority of persons as a scheme chimerical and impossible. We are impressed with a calm conviction that it does not even offer any real difficulty. The enormous labor of reconstruction involved, we seek not to deny or to underrate. But this is a trouble which must always be commensurate with the greatness of the reformation. This necessity would, however, most probably stimulate to the development and perfection of that most useful ally, the calculating machine. Rendered simpler in its construction by the very system which should require its services, and made popular and general by the new demand, it seems not improbable that a single century of the octonary empire would place the world on a higher platform than it would even reach without it. Such has been the usual history of difficulty and of success. A national government has but to *will* it to ensure its establishment ; and after the first impediments of custom were surmounted, we nothing doubt, that the facility and manifold conveniences of the new *regime* would form its most powerful support, and its surest recommendation to popular favor.

If the octonary system have the germ of vitality, here imagined, its adoption by any one of the great nations of Christendom would as surely pave the way to its universal prevalence, as did the introduction of the Hindoo notation, and of the Gregorian calendar. Nor are the obstacles which so long delayed those great reforms, either as numerous or as serious at the present day, as they were in by-gone centuries. The tone and temper of the times—intellectual, moral, and political—differ widely from those of our ancestors ; and in our common school system we have a moral mechanism for the inoculation of new truth, untried and unknown in all past ages.* Whenever the octonary numeration should be definitely established by political authority, we would immediately have all young children instructed for a year or two, only in the octonary arithmetic—as furnishing the easiest and most rational introduction to the knowledge of figures. And not until after a complete mastery of this arithmetic should they be taught the use of decimals—still required for a considerable period to enable reductions to be made from the old style to the new. This would be attended with no more labor than is the additional study now of ordinary Algebra ; while in the distinctive languages of the two scales would be found a safeguard against all danger or difficulty, in confounding the one value with the other.

* In the interesting report made to the Secretary of the Treasury, Dec. 30, 1856, by Prof. Bache, Superintendent of Weights and Measures, it is well remarked in relation to the facility of introducing a decimal system, that "One generation would nearly suffice to effect this change, if, as in Holland, the new weights and measures were introduced through the schools. The children of the country becoming familiar with them in the primary schools, seeing the actual material standards of length, capacity and weight at frequent and stated times in early youth, and retaining that familiarity as they passed into the higher schools, would be readily prepared for their universal use when reaching mature life."

The economy of time and labor which the system of octonary compu-
tation would infuse throughout the myriad commercial details daily en-
tering into the life of a busy and enterprising people, cannot be estimated,
and could not easily be exaggerated. The popular wonder would be no
smaller under the daily workings of this wiser system, that decimals
could have prevailed so many centuries—than is our wonder now that
the demands of trade could possibly have been satisfied by the awkward
and complex Roman scale of numeration.

The objections naturally brought against any disturbance of the exist-
ing order of accountancy (backed on the other hand by the indolent and
dilatory plea that we and our ancestors from earliest time have found it to
answer quite "well enough") are precisely those which have uniformly
opposed and retarded the introduction of every improvement. We are
informed by Sir John Bowring, in his interesting sketch of the Ex-
chequer system of England, that in quite recent times, Lord Granville
strongly resisted the abolition of the Latin phraseology, and the substitution
of the Hindoo numerals for the Roman, in the keeping of the public
accounts, on the ground that the continuance of the accustomed system
was necessary to preserve the comprehension of preceding records !*

The only question upon the subject that can be acknowledged as worthy
of discussion, is that which regards the beneficial character of the revolu-
tion. "Is, or is not, the change proposed a real improvement?" If it be
—if it be not only an improvement, but of all projected schemes the best—
then we assert the bolder logic—*its adoption is only a question of time!*
Prejudice, timidity or indolence, insensibility to the interest of the future,
or superstitious reverence for the gray-haired follies of the past, may each
or all oppose their ineffectual resistance ; they may indeed postpone for a
century or two the benefit to be enjoyed ; they may indeed throw in the
scale the added labor of accumulated work to be undone, but what is
"best" shall surely, in the end, secure its empire.

To the objection urged by some that the advantages to result are too
remote, and that even were the new arithmetic now inaugurated, the
present generation could not expect to have the full and peaceful enjoy-
ment of its alleged conveniences, we would reply that such has been the
case with every really great reform. The rewards of far-reaching bene-
factions are never for the present. We are in possession now of many

* "It is indeed scarcely credible, that the perplexing and entangled manner of keep-
ing accounts by the Roman numerals in the same barbarous style which was practiced
before the Norman Conquest, was maintained at the Exchequer almost down to the
present day ; and the introduction of the English language and the Arabic numerals
was successfully resisted by no less a personage than Lord Granville, on the ground that
if the barbarous usages of our ancestors were reformed, it would be difficult to under-
stand the accounts, and the records of departed time ; and hence he argued for the
necessity of perpetuating a system of complication, confusion and imperfection, not
on the common plea of the superior wisdom of our ancestors, but in full acknowledg-
ment and appreciation of the ignorance of the custom which was originally instituted,
and which had continued to reign triumphant among the Exchequer records" (*Bow-
ring's Decimal System, Chap. vii, page* 124).

priceless blessings whose first and feeble preparations were planned in former, unenjoying ages. Shall we reap the rich fruits grown from the unselfish providence of ancestral culture, and shall posterity be less favored? Patriotism and humanity reject the doubt. The octonary algorithm is pregnant with such great and widespread benefits—benefits to extend throughout all coming time, that its acquisition should be estimated as cheaply purchased by whole generations of transitional confusion.

The measure thus imperfectly advocated is by no means a new one. It is an incident of the highest interest and moment in the reign of that distinguished monarch, Charles XII of Sweden, that he not only contemplated the introduction of an octonary arithmetic, but that he commissioned Swedenborg (at that time celebrated for his scientific and mathematical attainments) to draw up the necessary details of the plan for establishing this system, together with an octonary scale of weights, measures and coins throughout his kingdom.* It appears that the premature death of the king very shortly afterward, alone prevented the consummation of this most sagacious and philosophic enterprise. But for this untoward circumstance this admirable mechanism would have thus been put into practical operation more than a century and a half ago ! Had it proved as successful as there is every reason to suppose it would, who can estimate the influence this engrafting would have had upon the present mathematical condition of Europe? Might we not now have been in the full and assured enjoyment of that happier system? The subject of this improved numerical notation had doubtless often occupied the minds of mathematicians long before this time, but this is probably the first occasion on which a deliberate and well-designed attempt was ever made to give it a practical existence and establishment. As such it is an event of no trivial importance, and must be regarded as ever memorable in the history of arithmetical reform.

In contemplating the practical working of this untried system, and forming an estimate of the character of the change required in the popular habits of thought, comparison and judgment, there can be no doubt that the octonary scale could be generally introduced with far greater facility, and made thoroughly familiar in a much shorter time, in its application to the divisions of money, weight and measure, than it could be in its more abstract application to the operations of universal numeration ; that in advance of the arithmetical reformation, it would be found highly expedient to introduce the simple and convenient system of weights and measures here proposed, as the best preparation for the successful introduction of the other.

Even were the octonary arithmetic (with all its own intrinsic excellences) not to be adopted, we still urge that these measures would be worthy of an independent establishment. After the variety of arithmetical reductions to which we are now accustomed under our present incongruous

* See note D, page 364.

tables, the uniform reduction of a single scale, which would alone be re-
quired in the new order, would give a very great simplification and relief,
and would in every probability be found upon the whole to entail less
inconvenience than that which would remain, with even the perfect deci-
malization of our various measures. So that even under the disadvan-
tages of a decimal dispensation there can be little doubt it could easily be
shown that our new system would still, in view of all the circumstances,
be the "best possible" one for popular use, and would most completely
furnish the elements of a perfect uniformity.

The system in use in this country has three units : The Yard, consist-
ing of 36 inches ; the Troy pound, consisting of 5760 grains, and the
Wine-gallon, containing 231 cubic inches ; these units being entirely inde-
pendent of each other. Upon these units our various tables of weights
and measures have been constructed without regard to regularity or fit-
ness for the practical purposes to which they must be applied, or without
any approach whatever to uniformity or similarity in the various multiples
or divisions of the units.

Any comprehensive and strictly philosophical system, as before stated,
can have but one unit, which must give law throughout. That unit will
most naturally be a linear measure, and whatever its derivation, where a
change is made, "the coincidences between the old and new ratios will
necessarily be rare. The best that can be done is to choose such a unit as
will produce the most of these."

In consideration of the strong desire of very many persons to retain our
present units, or at least the unit of measure, it is believed that the adop-
tion, as our standard, of the English *inch* or multiple of it, the inch being
the thirty-sixth part of the standard yard, which is also our standard yard,
with an octonary distribution of the various tables of weights, measures
and coins, although less philosophical and scientific than the plan just pro-
posed, would be much more readily accomplished. This would leave un-
disturbed all linear measures of Great Britain and of the United States,
and would possess all the essential elements for a successful adoption by
both countries.

A specified number of inches might be taken as the standard, and from
this all other measures, including those of surface, capacity and weight,
derived ; or if it should be considered preferable to retain the grain weight
instead of the linear unit, the side of a cube containing a weight of water
equal to a specific number of grains, might be taken as the standard.

The grain is a standard so widely used, and in medicine especially is
one of so great value as the exponent of so much knowledge and expe-
rience, that it should not be lightly set aside, and its surrender is a sacri-
fice which ought to be compensated by very undoubted advantages. So
far as medicine and pharmacy are concerned, it would seem to be the
most important unit to be preserved. Not only is it at present the recog-
nized measure of the physician and pharmacist throughout a great por-
tion of Europe, that in which chiefly is embodied the long acquired

experience and accumulated knowledge of the healing art, the laboriously ascertained and accurately observed relations and values of all the more active portion of the Materia Medica, but it is the measure which, outside of the medical and pharmaceutical professions, is the one almost universally employed as the unit of comparison for all minute investigations and precise determinations.

If either one should be adopted, the other would have to be abandoned ; and upon a careful consideration, notwithstanding the great importance of the *grain*, it is believed that the inch would be retained with less disturbance and with much greater advantage than the grain. Should the metric system be adopted, both the inch and the grain must be discarded.

Within a few years past various schemes have been proposed for promoting uniformity, but unless some one of them could be universally adopted, the confusion and complication would be increased instead of being diminished.

Prof. Oscar Oldberg has proposed for adoption by pharmacists and physicians, a new system based upon the " *Gramme* ;"* he proposes to divide the gramme into sixteen parts called " grains," thus making a new grain, a little smaller than our present grain ; four grammes to make a drachm, 8 drachms to make an ounce, and 16 ounces to make a pound ; the pound would thus consist of 8192 new grains, or about 7900 troy grains.

Even if this scheme should be adopted universally by pharmacists and physicians, which does not appear probable, it would but increase the difficulties under which we are now laboring ; it would only add one more to our already long list of tables of weights and measures to be learned.

There is no good reason why pharmacists or jewelers, or any other class of individuals, should have a special scale of weights and measures ; many of the evils experienced by them are those prevailing in all departments, and no improvements or reform can be either efficient or enduring which do not look to the welfare of the whole. It will be found impossible to give exclusive and confined attention to the weights and measures of any one profession ; there is absolute necessity of conformity among all the measures of trade and commerce, and of the reference of all to common laws and to a single standard.

These remarks will also apply to the scheme proposed by Mr. Wm. L. Turner for the use of pharmacists, published in the Proceedings of the Pennsylvania Pharmaceutical Association, 1886.

Mr. Turner proposes to divide the " *Gramme* " into 15 parts called " grains ;" to make the ounce equal to 500 of these grains, and the pound equal to 14 ounces, or equal to about 7200 troy grains.

Before attempting any change it should be well considered whether we have attained all the benefit within our reach, or whether at no greater cost we might not reap the advantages of a far more perfect system.

* Manual of Weights and Measures. By Oscar Oldberg, Pharm. D. Second edition. Chicago, 1887.

We would therefore propose to select for our "Module" a 16-inch rule instead of one of 15.672 inches, as suggested on page 338 ; all the tables as before given would remain unchanged in regard to their divisions and proportions, but of course the values would be slightly modified.

The Table of Measures of Capacity, and Weights, on page 355, shows the divisions and multiples of the "Modius" based upon this 16-inch Module, with their equivalents in Apothecaries' or Wine measure, and in cubic inches ; also the divisions and multiples of the "Pondus," with their corresponding Avoirdupois weights, and the connection between the measures and weights.

A great beauty resulting from the use of a cube number for a metrical radix, with octaval divisions, is shown by this table. It will be observed that the Modius and all of its multiples and divisions are *perfect cubes ;* and each one has a precise linear standard for the side of its cube ; thus, the Modius is the cube of the Module (or 16 inches); the Octa is the cube of 4 digits (or 8 inches) ; the Quart is the cube of 2 digits (or 4 inches) ; the Gill is the cube of 1 digit (or 2 inches); and so it is with every ascending or descending measure of capacity ; and the weight of the contents of these measures gives us a precisely corresponding series of weights.

To illustrate the contrasted awkwardness and complexity of a decimal system of measures, let the French "*Litre*" be selected. The *Litre* is the cube of the *decimetre*. Ten *litres* make one *dekalitre,* and if we would seek the cubic measure of this quantity, we shall find by a troublesome process of extracting the cube root, that 2 decimetres, 1 centimetre, 5 millimetres, and a decimal fraction .44347, and so on interminably, will give us an approximation to the length of the side, within an assignable limit of error. In other words, although there certainly is a cubic vessel, that shall contain exactly 10 *litres,* it is not within man's art of mensuration to tell precisely what the size of that cube must be. If, on the other hand, it were required to find the dimensions of a vessel holding exactly 8 *litres,* we know that a cube of 2 *decimetres* will give the measure with absolute precision ; or, if on the descending scale, it were required to find the size of a vessel holding exactly one-eighth of a *litre,* the cube of 5 *centimetres* gives us the perfect solution.

By the simple device of using multiples of one, two, and four times the size of such of these weights or measures as may be desirable, the use of fractions is entirely avoided, and a perfect system of weights and measures is supplied, by which any conceivable amount can be easily and accurately weighed or measured. Another beauty in our system is that it gives a maximum range of expression with the minimum number of pieces.

Of the weights in our table, those in ordinary use by the pharmacist, jeweler, etc., would be the *mite,* the *carat,* the *scrap,* the *semy,* and the *unce.* Weights of once, twice, and four times the quantity of each of these, or in all 15 weights, would enable us to weigh any possible quantity of *mites,* from one (which is less than half a grain) to 16170 grains ; that is to say, we could weigh 32760 different quantities ; these 15 weights

TABLE OF MEASURES OF CAPACITY, AND WEIGHTS.

Measures.		Wine Measure.					Cubes.	Cub. Inches	Weights.	Avoirdupois.		
		Galls.	Pts.	Oz.	Drs.	Minims.				Pounds	Oz.	Grains.
	1 Parvum					.519	$\frac{1}{8}$ ins.	$\frac{1}{512}$	1 Mite			.4936
8 Parvums	1 Morsel					4.15	$\frac{1}{4}$ "	$\frac{1}{64}$	1 Carat			3.949
8 Morsels	1 Ligule					33.2	$\frac{1}{2}$ "	$\frac{1}{8}$	1 Scrap			31.593
8 Ligules	1 Cup			4	4	26.	1 "	1	1 Semy			252.745
8 Cups	1 Gill			4	3	29.	2 "	8	1 Unce		4	271.96
8 Gills	1 Quart		2	3	3	59.	4 "	64	1 Libra	2	4	425.68
8 Quarts	1 Octa	2	1	11	7	54.	8 "	512	1 Stone	18	7	342.94
8 Octas	1 Modrus	17	5	15	7	18.	16 "	4096	1 Pondus	147	14	118.52
8 Modruses	1 Pipe	141	7	15	2	24.	32 "	32768	1 Ton	1183	2	73.16
8 Pipes	1 Butt	1135	7	10	3	12.	64 "	262144	1 Load	9465	1	147.78
8 Butts	1 Hold	9087	5	3	1	36.	198 "	2097152	1 Keel	75720	10	307.24

would take the place of the following 19 weights, which are now used to accomplish nearly an equivalent purpose, viz: $\frac{1}{2}$ grain, 1, 2, 3, 4, 5, 6, 10, 20, 30, 40, 60, 120, and 240 grains troy together with avoirdupois weights of 1, 2, 4, 8 and 16 ounces. These 19 weights make a total of 14104 grains, and would consequently be sufficient to weigh any number of half grains from 1 to 28208.

Upon examination of the above table, it will be seen that the *mite* is very nearly equal to half a grain, the difference being $\frac{6.4}{10.000}$, or about $\frac{1}{156}$th of a grain; *two mites* being about $\frac{1}{78}$th less than one grain; one *carat* is very nearly equal to 4 grains, being about $\frac{1}{20}$ grain less. One *scrap* is about $1\frac{1}{2}$ grains more than the half drachm. One *semy* is 34 grains more than half an ounce avoirdupois, or $12\frac{3}{4}$ grains more than half an ounce troy ; while four *unces* are equal to $18\frac{1}{2}$ ounces avoirdupois nearly.

Of the fluid measures the *ligule* is equal to half a fluid-drachm and 3.2 minims ; *two ligules* being 6.4 minims more than a fluid-drachm, or the medicinal teaspoonful ; the *cup* is equal to 4 fluid-ounces and $3\frac{1}{2}$ fluid-drachms ; 4 *gills* are equal to 1 pint and $1\frac{3}{4}$ fluid-ounces, and the new *quart* is equal to two pints and $3\frac{1}{2}$ fluid-ounces.

The smaller of these weights and measures assimilate so nearly with our present divisions, that for most practical purposes in medicine, pharmacy, etc., the difference would be inappreciable. It is true that all the valuable knowledge that clusters about the grain weight, in statistics of all kind, would have to be recalculated in the new weights, but as has before been stated this is a necessary consequence of *any* alteration in our unit.

If instead of retaining our linear unit, the inch, we had selected the grain weight, all of our weights would have been in even grains, while our measures would have been fractional quantities ; in this case, instead of taking the inch, we would take the length of one side of a cube of water weighing at its greatest density 256 grains ; such a cube would vary very slightly from a cubic inch ; its side would measure 1.004334 inches ; sixteen times this length would give us a " *Module* " equal to 16.069344 inches, and our " *Pondus* " would weigh 149 lbs., 12 oz. and 326 grains ; our " *scrap* " would be exactly 32 grains, our " *carat* " exactly 4 grains, and our " *mite* " exactly half a grain.

It is believed that the scheme here proposed, independently of its merits, would less disturb our present system of weights and measures than any that has yet been proposed, and would be, therefore, more easily intro-duced and willingly accepted.

And has not the time arrived in the general progress of commercial and international intercourse, and the rapid advance of our country in science, wealth and power, when her voice should be heard in an important mat-ter like this ! Should not our Congress invite all nations to appoint suit-able persons to be their representatives in a universal convention to be as-sembled for the purpose of devising and establishing a system of uniform weights and measures, practically applicable to the need and use of all peoples of the earth?

Such action could not fail to meet with a response due to the importance of the subject ; and if the great object be attained, to lead to results pro-ductive of vast and lasting benefit to the human race.

These suggestions are offered for the purpose of promoting discussion, investigation, and consideration of the subject in all its bearings, in the hope that when the time arrives in which a change must be made, and such a time will inevitably come, that a system may be adopted which has been, or can be demonstrated to possess the greatest advantages, and is admitted to be, of all schemes proposed, the truest, the wisest, the best.

Note A.

"The triumph of the art of calculation, and that to which mainly the modern system of numerical computation owes its perfection, consists in the 'device of place,' by which all necessity for distinguishing the nature of the units signified by any symbol is superseded. Like many other inventions of the highest utility this, when known, appears to arise so naturally and neces-sarily out of the exigencies of the case, that it must excite unqualified as-tonishment how it could have remained so long undiscovered. * * * That the honor of the invention of a system which produced such important effects as well on the investigations of science as in the common concerns of commerce, should be claimed by many contending nations, is what would naturally be expected. * * * All Arabian authors on arithmetic appear to agree that the first writer of that country upon this system of arithmetic was Mohammed ben Muza, the Khuwarezmite, who flourished about the year 900. This writer is celebrated for having introduced among his countrymen many important parts of the science of the Hindoos, to the cultivation of which he was devotedly attached ; and among other branches of knowledge thence derived, there is satisfactory evidence that this species of arithmetic was one. From the time of Mohammed ben Muza the figures and modes of calculation introduced by him were generally adopted by scientific writers of Arabia, although a much longer period elapsed before they got into com-mon popular use, even in that country. They were always distinguished by the name *Hindasi*, meaning the Indian mode of computation. * * * At the beginning of the eleventh century the use of the Arabic notation had be-come universal in all the scientific works of Arabian writers, and more espe-cially in their astronomical tables. The knowledge of it was of course com-municated to all those people with whom the Moors held that intercourse which would lead to a community of scientific research. In the beginning of the eleventh century the Moors were in possession of the southern part of Spain, where the sciences were then actively cultivated. In this way the use of the new arithmetic was received into Europe first in scientific treatises. A translation of Ptolemy was published in Spain in 1136, in which this notation was used ; and after this period it continued in general use for the purposes of science. Notwithstanding the knowledge and practice of this superior notation by scientific men, the Roman numerals continued to be used for purposes of business and commerce for nearly three centuries, and it was only by slow and gradual steps that the improved notation prevailed over its clumsy and incom-modious predecessor. The first attempt to introduce it for the purposes of com-merce was made by a Tuscan merchant, Leonardo Pisano, in 1202. Having

traveled in Barbary, he there learned the method of Hindoo arithmetic, and, struck with its superiority over that to which he had been accustomed, he determined that his countrymen should no longer be deprived of the benefits of it. He accordingly published his treatise in the Latin language ; in which he professes to deliver a complete doctrine of the numbers of the Indians. * * * A considerable period, however, was necessary to introduce this system into the common business of life. The extensive commerce maintained by the Italian States directed their attention to the subject at an earlier period than other nations ; and although, for scientific purposes, the date of the introduction of the Arabic numeration into Spain is earlier than that of its appearance in Italy, yet its use for the common business of life prevailed at a much earlier period among the Italian States than in any other nation of Europe" (*Lardner's Treatise on Arithmetic*, Book i, ch. ii).

The Hindoo numerals are found in various manuscripts of Italy bearing the dates 1212, 1220, 1228. But none are found in England till nearly two centuries later. Chaucer, the 'poet, who died in 1400, alludes to them in one of his poems as "*the figures newe.*"

According to Sir John Bowring ("Decimal System," pages 23–30), the first calendar in the English language in which the Hindoo numerals are employed, bears the date of "1431," and the earliest date known on a tombstone in these figures is "1454," the tombstone being that of "Elen Cook," in the church at Ware. The first English book which bears its date in these figures is the "*Rhetorica Nova*, Gulielmi de Saona, 1478." And in seals only one example has been found anterior to the sixteenth century, which bears the date 1484. "The Roman figures lingered longer in England," adds Bowring, "than in any other part of the European world, having found an asylum in the dark and dull regions of the Exchequer" (page 26). "It is indeed scarcely credible that the perplexing and entangled manner of keeping accounts by the Roman numerals, in the same barbarous style which was practised before the Norman Conquest, was maintained at the Exchequer almost down to the present day. * * * In addition to this strange and absurd system of Exchequer book-keeping, tallies continued to be used down to the year 1782. It was only in the year 1831 that the Committee on Public Accounts, of which I was the secretary, recommended the utter and complete abolition of the ancient system and the adoption of the Indian numerals. It was in consequence of this change that in the year 1835 the *tallies* were ordered to be burnt ; a conflagration which led to the destruction of both Houses of Parliament—the Exchequer in which the tallies were kept having formed a part of the ancient edifice of St. Stephen's" (*Sir John Bowring's Decimal System*, pages 124–125).

Delambre regards it as a fact humiliating to the pride of human genius that the discovery of the true notation of numbers by nine digits and zero should have escaped the sagacity of the illustrious geometers and mathematicians of ancient Greece. "The Hindoos," says Peacock, "consider this method of numeration as of divine origin. The invention of nine figures with the device of place being ascribed to the beneficent Creator of the universe. Of its great antiquity amongst them there can be no doubt, it having been used at a period certainly anterior to all existing records" (*Encyclopedia Metropolitana*). It can be traced back with certainty at least four centuries before its appearance among the Arabs, and as Lardner well re-

marks, since "none of these Hindoo authors claim either for themselves or
their predecessors the invention of this method of enumeration, but always
mention it as being received from the Deity, we may infer that it was prac-
tised in that country beyond the limits even of tradition." The Indian ori-
gin of our numerals being thus so well established, there is a manifest im-
propriety in continuing to designate them as the "Arabic figures," as is con-
stantly done in our school arithmetics. Let us give honor where honor is
due.

NOTE B.

It is remarkable that this binary system, according to the opinion of many,
was used in China, four thousand years ago, by Fohi, the founder of the
empire. A tablet of great but unknown antiquity, called the Cova of Fohi,
marked with a series of variously broken lines, and held in superstitious
reverence by the Chinese, as containing the mystery of a divine wisdom, has
been found to be completely deciphered by the notation of binary arithmetic.
When Leibnitz had extensively circulated his scheme or invention through
the various scientific journals, and by means of his own correspondence—it
appears to have found its way even to China, and to have attracted the
attention of a Jesuit missionary at Pekin, named Bouvet. This ecclesiastic,
engaged at the time in the study of the Chinese antiquities, discovered and
immediately communicated to Leibnitz, with much exultation and enthusi-
asm, the surprising fact that his system furnished a perfect key to the mys-
terious lines upon the ancient Cova—hitherto inscrutable, or interpreted only
by the speculations of the most extravagant mysticism. The lines of Fohi
are arranged in an octagonal form, so as to make the ends approach ; each
set of the eight series being disposed on a side of the octagon.

These lines transferred from the Cova tablet, and placed in a straight line,
are here represented. The row of figures in front expresses the value of each
compound symbol, the other figures, which represent
the binary notation, manifestly exhibiting a perfect
correspondence with the symbols throughout.

"These figures of eight cova," says Mr. Peacock,
(in the *Encyclopedia Metropolitana*), "are held in
great veneration, being suspended in all their temples,
and though not understood, are supposed to conceal
great mysteries, and the true principles of all philoso-
phy, both human and divine."

This inscription is exceedingly interesting as ex-
hibiting a true example of that philosophic notation,
the device of the cipher—and the determination of
value by place. The absence of any other traces of
such a notation in China, and its well-known anti-
quity in India, where it had been so fully elaborated,
would lead to the suspicion that it was to this latter
country that Fohi was indebted for this curious record
of ingenious thought. It appears that Bouvet was
fortunate enough to find, subsequently, a Great Cova,
in which these markings were carried to a period
eight times the extent of the Small Cova. In the *Edinburgh Encyclopedia*

0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

(Article " Arithmetic "), it is stated in reference to this subject, that Father Bouvet, who first suggested this explanation and communicated it to Leibnitz, afterward procured, during his residence in China, the *Great Figure of Fohi*, which extends as far as 64. The exact coincidences which he still found to prevail between the combinations of these lines and the figures of the binary notation, left no doubt with regard to the justness of his conjecture; and we cannot help remarking that the restitution of the true sense of those characters, after so long an interval of time, is a very singular fact in the history of science.

Note C.

It is interesting to trace the history of the gradual development, in modern times, of the grand but difficult project of obtaining from nature a constant and universal standard of length. It is obvious that no such objects of ultimate reference as the human foot, or arm, or cubit, or as "thirty-six barley corns round and dry," can be regarded as natural standards, since they are wholly useless for the purpose of any precise determination. And all measures derived from them are purely arbitrary, as their authority is obtained from positive enactment, merely, and not from any agreement with their nominal originals. Hence it is not at all surprising that "cubits" and "feet" come to signify anything the civil power may enact; the former of these denominations ranging through every gradation of value, from the *covid* of 14½ inches to the royal Egyptian *cubit* of 25½ inches, and the latter from the Pythic foot of 9¾ inches, to the Geneva foot of 19 inches. Nor would it ever be possible from such sources, to reproduce a lost standard, with even the rudest approach to exactness. As Mr. Adams has well remarked, "For all the uses of weights and measures in their ordinary application to agriculture, traffic, and the mechanic arts, it is perfectly immaterial what the natural standard to which they are referable may be. The foot of Hercules, the arm of Henry the First, or the barley-corn is as sufficient for the purpose as the pendulum, or the quadrant of the meridian " (*Report to Congress*).

"The first attempt at fixing such a standard as should be accurate and universal, both as to place and time, is due to the inventive genius of the celebrated Huyghens. That philosopher demonstrated that the times of the vibrations of pendulums depend on their length only. * * * Hence he conceived that the pendulum might afford a standard or unit for measures of length " (*Edinburgh Review*, Vol. ix, page 373). It was in his "Horologium Oscillatorium" (published about 1670), that Huyghens proposed the use of the seconds' pendulum as a universal and perpetual measure ; this length to be divided into three equal parts ; and this third part (about 13 inches) to be called the *horary foot.*

The celebrated Picard, who first measured from Paris to Amiens in 1669, an arc of the meridian in France, making the degree equal to 68.945 miles (a measurement memorable as having furnished Newton with the means of verifying his grand theory, incapable of determination from the pre-existing data), also proposed in 1671, in agreement with the idea of Huyghens, that the pendulum beating seconds should be adopted as the unit of length. Picard has the merit of having first thrown out the suggestion that the diurnal rotation of the earth ought to affect the oscillation of the pendulum, and

that it ought to vibrate more rapidly toward the poles than toward the equator. He accordingly tried the pendulum at Uranibourg, at Paris and at Cette, but was not fortunate enough to discover any sensible difference. Roemer also found the length the same at London.

Richer, however, in the same year, 1671, or early in 1672, while engaged in the duties of his commission at Cayenne, on observing the length of the seconds' pendulum at this place (lat. $4^\circ 56'$ north of the equator), found it sensibly shorter than at Paris ($48^\circ 50'$ north), the difference being about a line and a quarter. Richer's discovery that the pendulum varied in length with the latitude, deprived it of that uniform character considered so necessary in a linear *standard*.

The Abbe Gabriel Mouton, a distinguished mathematician who flourished at the same time, appears to be the first who suggested a measure derived from the earth. He proposed, almost simultaneously with the publication of Huyghens, a *decimal system of measures* based on the value of a minute of arc, as derived from Riccioli's length of a degree. This minute of the degree he called a *miliare*, the thousandth part of which he called a *virga*, equal to 5 feet $4\frac{1}{5}$ inches. We have here the germ of the present French metrology.

Cassini, who in 1718 repeated the measurements of a meridian made by Picard (extending his arc, however, further south, namely, from Paris to Dunkirk, and making the degree 69.119 miles), proposed the earth's radius as the unit of length. He afterward in his book, " *De la Grandeur de la Terre*," proposed as a unit the six-thousandth part of a minute of a degree of a great circle of the earth, a measure very nearly equal to the foot.

In 1748 M. de la Condamine (who had recently returned from measuring a degree at the equator in Peru), in a memoir read before the Academy of Sciences, resumed the idea of the pendulum as the unit of length, proposing that it should be taken as beating seconds at the equator, as the most notable line of latitude, and as one likely to avoid all the prejudices which might arise from national jealousy were the latitude of any particular place selected. We see from this the anxiety felt to secure a standard which might be common and uniform among nations.

On the 15th of January, 1790, in accordance with President Washington's recommendation, the House of Representatives

"Ordered, That it be referred to the Secretary of State to prepare and report to this House, in like manner, a proper plan or plans for establishing uniformity in the currency, weights and measures of the United States."

On the 15th of July of that year the House of Representatives received from the Secretary of State (Mr. Jefferson) his report of the proper plan for establishing the desired uniformity, as requested by the House.

In this elaborate report the Secretary proposed " that the standard of measure be a uniform, cylindrical rod of iron of such length as, in latitude 45°, in the level of the ocean, and in a cellar or other place, the temperature of which does not vary through the year, shall perform its vibrations in uniform and equal arcs in one second of mean time."

Starting from this standard, he proposes two distinct plans for the consideration of the House, that they might, at their will, adopt the one or the other exclusively, or the one for the present and the other for the future time, when the public mind may be supposed to have become familiarized to it.

The first plan was *to define and render uniform and stable the existing sys-*

tem ; to make the foot to bear a definite ratio to the standard pendulum rod ; to reduce the dry and liquid measures to corresponding capacities by establishing a single gallon of 270 cubic inches, and a bushel to be equal to eight (8) gallons, or 2,160 inches—that is, to one and one-fourth cubic feet ; to make the ounce to be the weight of one-thousandth part of a cubic foot of water ; to retain the more known terms of the two kinds of weights in use, reduced to one series ; and to express the quantity of pure silver in the dollar in parts of the weight so defined.

The second plan was to reduce "every branch to the same decimal ratio already established in coins, and thus bring the calculation of the principal affairs of life within the arithmetic of every man who can multiply and divide plain numbers."

Except in the length of the fundamental unit, and in the nomenclature, this was essentially that of the metrical system of France.

These two plans were sharply opposed to each other, and it was to be expected that the desire for a decimal division, and symmetry of system on the one hand, and the reluctance to make a violent change on the other, should elicit no little discussion.

This report was communicated to the Senate in December of that year and referred to a committee. That committee reported on the 1st of March, 1791, that, "as a proposition has been made to the National Assembly of France for obtaining a standard of measure which shall be invariable, and communicable to all nations and at all times ; as a similar proposition has been submitted to the British Parliament in their last session ; as the avowed object of these is to introduce an uniformity in the measures and weights of the commercial nations ; as a coincidence of regulation by the Government of the United States on so interesting a subject would be desirable, your committee are of opinion that it would not be eligible, at present, to introduce any alteration in the measures and weights which are now used in the United States." This report was adopted.

In 1790, Talleyrand proposed to the constituent Assembly of France, that in view of the great diversity and confusion in the weights and measures of the country, a commission should be appointed for the purpose of consulting with a similar commission from the English Government, upon the subject of establishing a uniform international system of metrology, founded upon a single and universal standard. The proposal alluded to the only two natural standards which presented themselves, viz., the measure of the earth and the pendulum, and expressed a decided preference for the latter. The result of this movement was the appointment of Borda, Lagrange, Laplace, Monge, and Condorcet, as commissioners to examine into and report upon the subject. After a careful consideration of the three plans submitted, namely, the pendulum, a quarter of the equator, and a quarter of the terrestrial meridian, they very judiciously agreed in decidedly recommending the latter ; regarding the pendulum as an unsuitable standard, whether taken at forty-five degrees of latitude or at the equator.

The attempt to enlist the co-operation of England proved abortive. "The operation of changes of opinion there," says Mr. Adams, "is slow—the aversion to all innovations deep. More than two hundred years had elapsed from the Gregorian reformation of the calendar, before it was adopted in England. ＊ ＊ ＊ After a succession of more than sixty years of inquiries and

experiments, the British parliament have not yet acted in the form of law "
(*Report to Congress*).

Just five hundred years after the statute of 17th Edward II (A.D. 1324),
enacted that "three barley-corns round and dry, make an inch—twelve inches
make a foot," etc., the first change was made in the legal definition of the
foot. By the act of 5th George IV, c. 74 (1824), it is declared "the standard
yard is the distance between the centres of the two points on the gold studs
in the straight brass rod now in the custody of the Clerk of the House of Com-
mons, whereon is engraved 'Standard yard, 1760,' the brass being at the tem-
perature of 62 degrees of Fahrenheit's thermometer." "The Yard, if lost,
defaced, or otherwise injured, may be restored by comparing it with the pen-
dulum vibrating seconds of mean time in the latitude of London, in a vacu-
um, on the level of the sea, the yard being in the proportion of 36 inches to
39.1393 inches of the pendulum." This was the first attempt to refer the
English foot to a natural standard.

Ten years afterward, or in 1834, the contingency provided for by this sta-
tute actually occurred by the burning of the Houses of Parliament ; in which
conflagration the celebrated brass standard of Bird was destroyed. Although
the only actual legal standard was thus lost, no attempt was made to restore
it by the pendulum, as provided by law ; but the mean of several different
standards, including one belonging to the Royal Astronomical Society (for-
tunately the Astronomical Society had procured a most carefully prepared
copy of the imperial standard yard, and the Mint was in possession of an
exact copy of the pound), was selected as giving the nearest approximation
to the legal standard yard.

A commission was appointed by the British Government, in 1838, "to con-
sider the steps to be taken for the restoration of the Standards of Weight
and Measure." The commissioners in their report, made in 1841, say : "We
are of opinion that the definition contained in the Act 5, Geo. IV, c. 74, ss. 1
and 4, by which the standard yard and pound are declared to be respectively,
a certain brass rod and a certain brass weight therein specified, is the best
which it is possible to adopt. Since the passing of the said act, it has been
ascertained that several elements of reduction of the pendulum experiments
therein referred to are doubtful or erroneous; thus the reduction for the
weight of air was erroneous ; the specific gravity of the pendulum was
erroneously stated, the faults of the agate plates introduced some degree of
doubt, and sensible errors were introduced in the operation of comparing the
length of the pendulum with Shuckburgh's scale, used as the representative
of the legal standard. It is evident, therefore, that the course prescribed by
the act would not necessarily reproduce the length of the original yard.
Several measures however exist, which were most accurately compared with
the former standard yard. And we are fully persuaded that, with reasonable
precautions, it will always be possible to provide for the accurate restoration
by means of material copies which have been carefully compared with them,
more surely than by reference to any experiments referring to natural con-
stants." And the report concludes by recommending "that the standard of
length be defined by the whole length of a certain piece of metal or other
durable substance, supported in a certain manner, at a certain temperature ;
or by the distance between two points or lines engraved upon the surface of
a certain piece of metal or other durable substance, supported in a certain

manner and at a certain temperature; but that the standard be in no way defined by reference to any natural basis, such as the length of the pendulum vibrating seconds in a specified place. * * * That the standard of weight be defined by a certain piece of metal or other durable substance," etc.

It thus appears as the result of this last commission in England, that the people of that country are disposed to abandon all attempts at obtaining a natural standard, and to recur to the authority of an arbitrary rod or piece of metal, whose length has been derived from prescriptive custom. It should be considered, however, that after a natural standard has been obtained, we still have all the means of its material perpetuation, suggested in the commissioners' report. And no foreign community is ever likely to accept as an authoritative unit of measure, a certain brass rod manufactured in England, and incapable of any more precise definition.

Mr. Baily was selected to prepare the new standard, having five copies of the preceding on which to base his comparison; on his death, in 1844, Mr. Sheepshanks continued the necessary observations. Of several standard copies finally prepared by him, each being a square inch bar, of a bronze consisting of copper with a small percentage of tin and zinc, 38 inches in length, with half inch wells sunk to the middle of the bar, one inch from each end, in which the lines defining the yard are drawn on gold plugs—six were finally selected and reported by the commissioners in March, 1854. Of these, the one marked "Bronze 19" was selected as the parliamentary standard yard, the remaining five being deposited, along with copies of the standard of weight, with as many public institutions and scientific bodies. These standards were legalized in July, 1855; and in case of loss of the parliamentary copy, it was provided that the standard should be restored by comparison of the other selected copies, or such as might be available.

Bronze bar No. 11 which has the standard length at a temperature of 61.79° has been presented to the United States, and is the actual standard of comparison.

In addition to the difficulties of obtaining from the pendulum the reconstruction of a lost standard, as above indicated, it is not unimportant to note that there is an original uncertainty in the determination of its length, of nearly the thousandth part of an inch. "We cannot venture to say that the clock's rate in a given day, can be determined certainly to within one-tenth part of a second, although the comparisons have been made at an interval of 24 hours. Seeing then that the *free* pendulum is compared with the clock only over a small fraction of the day, it is a great deal to expect that *its* daily rate can be ascertained to within one second of time. A change of one second per day in the rate of a clock, corresponds to a change of $\frac{1}{43200}$, in the length of the pendulum, which is about $\frac{1}{1100}$ of an inch, or $\frac{1}{43}$ of a millimetre; and therefore we may regard this distance as indicating the probable limit of exactitude" (*Encyclopedia Britannica*, 8th edition, Vol. xvii, page 384, article "Pendulum," by Edward Sang).

NOTE D.

The only account we have been able to obtain of the important movement of Charles XII toward superseding the decimal by the octonary system, throughout Sweden, is that contained in a volume entitled "A Compendium

of the Theological and Spiritual Writings of Emanuel Swedenborg " (royal octavo), published at Boston by Crosby & Nichols, 1854. In the life of Swedenborg, prefixed to the " Compendium," it is said : " In 1719 he published four works ; first, ' *A Proposal for fixing the value of Coins and determining the Measures of Sweden, so as to suppress fractions, and facilitate Calculations.*' After which he was commanded by his Sovereign to draw up an Octonary Computus (a mode of computing by eighths), which he completed in a few days, with its application to the received divisions of Coins, Weights, and Measures ; a disquisition on Cubes and Squares, and a new and easy way of extracting Roots ; all illustrated by appropriate examples " (Life, p. 9). As Swedenborg devised for his " Octonary Computus," both a set of characters and of new names, we were exceedingly anxious to have enriched this Paper with their representation. We have failed, however, to find any clue to these early publications in any of the public libraries or private collections to which we have had access. The only additional reference to the subject in the volume above referred to, is contained in a letter from Swedenborg to M. Nordberg, written after the death of Charles XII, which appears to detail the monarch's first conception of the project of a reformation in the popular system of numeration. An extract giving all that relates to the subject of octonary computation, is here copied :

Letter of M. Swedenborg, Assessor of the Board of Mines, to M. Nordberg, Author of the History of Charles XII.

"SIR :—As you are now actually engaged upon the Life of Charles XII, I avail myself of the opportunity to give you some information concerning that monarch, which is perhaps new to you, and worthy of being transmitted to posterity. * * * Conversing one day with the King upon arithmetic, and the mode of counting, we observed that almost all nations, upon reaching ten, began again ; that those figures which occupy the first place, never change their value, while those in the second place were multiplied ten-fold, and so on with the others ; to which we added that men had apparently begun by counting their fingers, and that this method was still practised by the people ; that arithmetic having been formed into a science, figures had been invented which were of the utmost service ; and, nevertheless, that the ancient mode of counting had been always retained, in beginning again after arriving at ten, and which is observed by putting each figure in its proper place.

The King was of opinion that had such not been the origin of our mode of counting, a much better and more geometrical method might have been invented, and one which would have been of great utility in calculations, by making choice of some other periodical number than 10. That the number 10 had this great and necessary inconvenience, that when divided by 2, it could not be reduced to the number 1, without entering into fractions. Besides, as it comprehends neither the square, nor the cube, nor the fourth power of any number, many difficulties arise in numerical calculations. Whereas, had the periodical number been 8, or 16, a great facility would have resulted, the first being a cube number of which the root is 2, and the second a square number of which the root is 4 ; and that these numbers being divided by 2, their primitive, the number 1 would be obtained, which would be highly

useful with regard to money and measures, by avoiding a quantity of frac-
tions. The King, after speaking at great length on this subject, expressed a
desire that we should make a trial with some other number than 10. Having
represented to him that this could not be done unless we invented new figures,
to which also names altogether different from the ancient ones must be given,
as otherwise great confusion would arise, he desired us to prepare an example
in point. We chose the number 8, of which the cube root is 2, and which
being divided by 2, is reduced to the primitive number 1. We also invented
new figures, to which we gave new names, and proceeded according to the
ordinary method ; after which we applied them to the cubic calculations, as
well as to money, and to measures. The essay having been presented to the
King, he was pleased with it" (*Appendix to Life, etc.*, pp. 123, 124).

On the so-called Alaguilac Language of Guatemala.

By D. G. Brinton, M.D.

(Read before the American Philosophical Society, Nov. 4, 1887.)

In his valuable treatise on the ethnography of the Republic
Guatemala, Dr. Otto Stoll classes the Alaguilac language, once
spoken by a tribe resident on the Motagua river in that country,
among the languages of unknown affinities, *Sprachen unbekann-
ter Stellung ;* and he also adds, that at the time of his visit to
the vicinity—now about five years ago—the tongue was entirely
extinct, being supplanted by the Spanish.*

It were greatly to be regretted that any language or dialect
should perish completely, leaving no record behind it by which
we can assign its place in the linguistic scheme. I am happy to
say, this is not the case with the Alaguilac. I have in my hands
materials from several sources from which to identify this now
extinct tongue, and also to cast some interesting glimpses on the
ancient civilization of the tribe which once spoke it. These
sources are :—

I. Four leaves in folio, originals, from the archives of the
Parish of San Cristobal Acasaguastlan, dating from 1610 to
1637, in bad condition, but mostly legible.

II. A collection of words and phrases obtained in 1878 by
Francisco Bromowicz from an Indian woman at the village of

* Stoll, *Zur Ethnographie der Republik Guatemala,* s. 172. Also, *Guatemala, Reisen und
Schilderungen,* s. 304.